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Abstract

This paper studies the dynamic evolution of an economy in which parents can choose to send their child to a public or private school and vote over taxes used to fund public schools. The objective is to study growth and the evolution of income distribution in a model where alternative education systems coexist. In the model studied, the endogenous distribution of income is bimodal and cannot be fully analytically characterized. Equilibrium is characterized and the dynamics studied analytically. Simulations of the model calibrated to US data are used to complement this analysis. A bimodal income distribution based on education emerges. Public education students converge to a low income equilibrium while private education students experience endogenous growth and have higher incomes. However, public education students also experience long run growth through a spillover from the growth experienced by private education students. The model identifies possible problems with the existence of a private alternative to public education, such as the emergence of a education based class structure. However, such an institutional setting can raise incomes and growth relative to a compulsory public education system while still reducing income inequality.

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1 Introduction

Most countries have some form of public education, justified on either efficiency or equity grounds or, in some cases, both. Another common feature is that wherever we find public schooling there is usually a private alternative. This institutional setting generates a wide range of issues. The private alternative is valued for the choices it provides, for example, pedagogical or religious, and for the competition it provides to the public schools. However, the same private education alternatives are criticised for being exclusive, perpetuating priviledge and income inequality. There is never a shortage of opinions on which is better and why. The interaction between public and private education raises questions about whether the exit of students from public into private schools raises the quality of public education or results in lower taxes and lower quality public education? In a dynamic framework it can be asked if spillovers between public and private education sectors exist or does the private education alternative lead to the evolution of a well educated elite? These issues are of interest to education policy makers and parents everywhere and the present study sheds some light on them.

This paper sets out to study growth and the evolution of income distribution in an economy where parents can choose to send their child to either a public or private school. Public schools are financed by a tax that is determined by a majority vote of all parents, thus generating a political tension between parents of public and private education students. Numerous examples of dynamic analyses of public and private education exist in the literature. Typically these studies focus on comparisons of separate public and private education models, such as in Glomm and Ravikumar (1992), Gradstein and Justman (1997) and Saint Paul and Verdier (1993). Another common theme found in the literature is the case where public and private education are complements in human capital production, as in Bénabou (1996), Eckstein and Zilcha (1994) and Kaganovich and Zilcha (1999). The model in this paper departs from the literature by analysing the case where both public and private education coexist as mutually exclusive alternatives rather than comparing public with private education.

The paper builds on the static analyses of public and private education choice which include Epple and Romano (1996), Glomm and Ravikumar (1998a), Hoyt and Lee (1998) and the seminal work of Stiglitz (1974).[1] These models of education choice are augmented with an endogenous growth model driven by education and human capital accumulation. The dynamics resemble the spillover models of Bénabou (1996), Glomm and Ravikumar (1992), Lucas (1988) and Tamura (1991) while the coexistence of two education alternatives leads to multiple equilibria resembling results in Galor and Zeira (1993).

Results from Epple and Romano (1996) are applied to identify voting equilibria in each generation. It is established that the median income voter is decisive and, with sufficient positive skewness in the income distribution, an equilibrium with both public and private education exists. Two voting coalitions are identified, those with low incomes are in favor of public education and vote for non-zero tax rates while those with high incomes prefer private education and vote for a tax rate of zero.

Given a voting equilibrium, the dynamics of the model are analysed and multiple equilibria are identified. The co-existence of public and private education leads to polarisation in the income distribution. Public education students converge to a low income equilibrium while private education students experience endogenous growth and have higher incomes. These low and high income groups evolve endogenously and resemble convergence clubs; see Galor (1996). Whether a family ends up in a low or high income equilibrium depends on initial conditions. If the first generation of a family is poor then the family will attend public school, converge to the low income equilibrium and will be unable to break out of the education poverty trap as it is assumed that capital markets for human capital investment do not exist. The dependence of public education on average human capital leads to a fiscal growth spillover. The endogenous growth experienced by private education students raises average human capital and drives long run growth in public education incomes. Another benefit of the mixed education model is that initially, the poor experience faster growth than they would under the private alternative. This is because public provision provides the poorest with much greater education expenditures.

The endogenous income distribution is studied using numerical simulations with the model calibrated to recent US income distribution and public education data. The simu-

[1] Also see the work by Epple and Romano (1998) and Nechyba (1999, 2000) for static education choice models with peer effects and Durlauf (1994) and Fernandez and Rogerson (1996) for multi-district models of education.

lations identify a bimodal income distribution, though over time the upper income private education group shrinks.

The mixed education model is compared to public and private education benchmark models and it is found that the mixed education model has strong properties of reducing income inequality, like the public education benchmark, while generating higher levels of per capita income than the public education benchmark. Income lies between that of the public and private education benchmark models with the fiscal growth spillover keeping mixed education per capita income above that of the public education benchmark model. The model's sensitivity to the initial distribution of income is tested by assuming higher inequality and positive skewness. It is found that inequality is beneficial for growth, more students can afford to stay in private schools, generating a larger growth spillover and higher overall growth. This comes at the cost of higher income inequality. Sensitivity analysis also shows that the mixed education model can be unstable with parents switching out of the private system over time and in some cases, the private system closing down completely in the long run.

The paper proceeds as follows. Section 2 sets out the model of education choice. The equilibrium properties of the model are studied in Section 3. Political equilibria are investigated and conditions for their existence are established in Section 4. The dynamic evolution of income distribution is investigated in Section 5, where a special case of the model is characterised analytically and the benchmark public and private education models are outlined. Numerical simulations along with sensitivity analysis of the model are presented in Section 6. Section 7 provides a summary and conclusions.

2 A Model of Education Choice

An overlapping generations framework with a continuum of two period lived agents is employed. These agents accumulate human capital when young and supply labour when old. Old agents each have one child, maintaining a constant population which is normalised to unity. It is assumed that old agents gain utility from providing education to their child and have a choice between public education and private education, these educational alternatives are assumed to coexist. Old agents choose which system their child is to attend but irrespective of this choice, pay income taxes which are used to finance public education. The rate at which incomes are taxed is endogenously determined through a majority vote by the cohort of old agents in each period. Heterogeneity is introduced to the model through the human capital endowments of the first generation. Arguments in the utility function include the parents education expenditure on the child ($q_{i,t+1}$) and consumption of a composite commodity when old ($c_{i,t+1}$).

Agent i born in period t solves the following problem:

$$\max_{\{c_{i,t+1},q_{i,t+1}\}} \ln(c_{i,t+1}) + a\ln(q_{i,t+1}) \tag{1}$$

subject to the respective education choice, human capital production function, income identity and individual budget constraint:

$$q_{i,t+1} = \begin{cases} e_{i,t+1} & \text{if } e_{i,t+1} > 0\text{; private education chosen.} \\ E_{t+1} & \text{if } e_{i,t+1} = 0\text{; public education chosen.} \end{cases} \tag{2}$$

$$h_{i,t+1} = \theta h_{i,t}^{\gamma} q_{i,t}^{1-\gamma} \tag{3}$$

$$y_{i,t+1} = h_{i,t+1} \tag{4}$$

$$c_{i,t+1} = (1-\tau_{t+1})\, y_{i,t+1} - e_{i,t+1} \tag{5}$$

while the government budget constraint is given by:

$$E_{t+1} = \tau_{t+1} \int_{\mathbf{H}} h f_{t+1}(h)\, dh / P_{t+1} = \tau_{t+1} H_{t+1} / P_{t+1} \tag{6}$$

with the parents education expenditure, $q_{i,t}$, and human capital, $h_{i,t}$, treated as given. Agents also treat the political outcome, the equilibrium public education choice, (E_{t+1}, τ_{t+1}), as given. The coefficient a in equation (1), measures the importance of education expenditure relative to private consumption to the parent, it is assumed $a \in (0,1)$. The specific value of a is chosen through the calibration of the model to income distribution and education expenditure data.

The human capital endowments of the first old generation, $h_{i,0}$, are distributed with probability density function $f_0(h)$. The human capital distribution of subsequent generations is endogenous, determined by (3) and denoted by $f_t(h)$, with support $\mathbf{H} \subset [0,\infty)$.

Equation (2) identifies the agent's choice of education system for their child, with $e_{i,t+1}$ denoting private education expenditure and E_{t+1} denoting public education expenditure.

An agent choosing $q_{i,t+1} = e_{i,t+1} > 0$, sends its child to the private education system. Alternatively, choosing zero private education expenditures, $e_{i,t+1} = 0$, means the parent is choosing public education for the child, $q_{i,t+1} = E_{t+1}$.

The human capital accumulation process is described by equation (3). It is assumed that $\gamma \in (0,1)$ to ensure diminishing returns to each input while the scale parameter $\theta > 0$. The inputs to the education process include, the parents education expenditure on the child, $q_{i,t}$, and the parents human capital, $h_{i,t}$. The assumption that the exponents on education expenditure and parental human capital sum to unity focuses the analysis on endogenous growth.[2]

The assumption that income earned when old, $y_{i,t+1}$, is given by human capital, $h_{i,t+1}$, is embodied in equation (4). The implication is that all agents supply one unit of time and the wage rate is given by the level of human capital. This provides a clearer focus on education choices and their effects on income distribution, abstracting away from labour supply choices of old agents. Equation (5) is the individual's budget constraint. Consumption, $c_{i,t+1}$, is limited by after tax income less any private education expenditures on the agent's child.

Per student public education expenditures are given by equation (6) and depend on the tax rate, per-capita income, given by $H_{t+1} = \int_{\mathbf{H}} h f_{t+1}(h)\, dh$, and the proportion of the population choosing to consume public education, given by P_{t+1}. Normalising the population to unity leaves H_{t+1} representing both aggregate and per-capita human capital and income.

3 Equilibrium

A general equilibrium for the model described by equations (1) to (6), is given by the set of sequences of individual choices $\{c_{i,t}, q_{i,t},\}_{t=0}^{\infty}$, public education outcomes $\{(E_t, \tau_t), P_t\}_{t=0}^{\infty}$ and distributions of human capital $\{f_t(\cdot)\}_{t=0}^{\infty}$, such that:

(i) The individual choices $c_{i,t}$and $q_{i,t}$ maximise utility of agent i who is old in period $t \geq 0$, given the parent's human capital $h_{i,t-1}$ and education expenditure $q_{i,t-1}$ while also treating the public education choices (E_t, τ_t) and P_t as given.[3]

[2]For an analysis of the neoclassical growth case, where the sum of the exponents is less than unity, see Cardak (2001).

[3]The first cohort of old agents are born old in period $t = 0$, with human capital endowments $h_{i,0}$ given by the distribution function $f_0(\cdot)$. These agent's incomes are exogenous rather than being given by equation

(ii) Individual human capital, $h_{i,t+1}$, and its distribution $f_{t+1}(\cdot)$ are given by equation (3) for agents born in period $t \geq 0$.

(iii) Given the individual choices of $q_{i,t}$ and the aggregation of these choices into P_t, given by equation (15) below, the public education choice, (E_t, τ_t), is preferred to all other feasible policies by a majority of old agents in period $t \geq 0$.

The discrete nature of the education choice requires a solution technique where optimal choices under both alternatives are determined and compared. The education alternative providing greatest utility is then chosen.

In period $t+1$, the income of an agent $(y_{i,t+1})$ is determined by the human capital of the parent $(h_{i,t})$ and the education provided to the agent when it was young $(q_{i,t})$:

$$y_{i,t+1} = h_{i,t+1} = \theta h_{i,t}^{\gamma} q_{i,t}^{1-\gamma} \tag{7}$$

This income is independent of any choices the agent may make, including the education choice it makes for its own child.

An agent who chooses $q_{i,t+1} = e_{i,t+1} > 0$ is providing private education for its child. The model is solved by allocating income between education and consumption in the following way:

$$c_{i,t+1}^{r} = \frac{(1-\tau_{t+1})\, y_{i,t+1}}{(1+a)} \tag{8}$$

$$q_{i,t+1} = e_{i,t+1} = \frac{a\,(1-\tau_{t+1})\, y_{i,t+1}}{(1+a)} \tag{9}$$

where the r superscript denotes choices of private education consumers.

The type of education chosen for the child, $q_{i,t+1}$, need not be the same as that provided to the parent when young, $q_{i,t}$. A parent who received private education when young may provide public education to its child or vice versa. Whether such switches occur is studied below.

The utility of an agent providing private education for its child is given by:

$$U\left(c_{i,t+1}^{r}, q_{i,t+1}\right) = \ln\left(c_{i,t+1}^{r}\right) + a \ln\left(e_{i,t+1}\right) = V^{r}\left(\tau_{t+1}; y_{i,t+1}\right) \tag{10}$$

(3).

where indirect utility V^r is a function of taxes for a given level of human capital, which can be seen by substituting equations (8) and (9). Increases in the tax rate decrease disposable income while providing nothing in return to private education consumers, thus the function V^r is monotone decreasing in the tax rate, τ_{t+1}, and unaffected by the level of public education expenditures, E_{t+1}.

An agent providing public education to its child consumes all post tax income and chooses $e_{i,t+1} = 0$, allowing $q_{i,t+1} = E_{t+1}$, treating the public education enrollment, P_{t+1}, and choices, (E_{t+1}, τ_{t+1}), as given. When public education is provided, the optimal level of consumption is:

$$c^u_{i,t+1} = (1 - \tau_{t+1})\, y_{i,t+1} \tag{11}$$

The utility of an agent choosing to provide public education to its child is given by:

$$U\left(c^u_{i,t+1}, E_{t+1}\right) = \ln\left(c^u_{i,t+1}\right) + a\ln\left(E_{t+1}\right) = V^u\left(E_{t+1,}\tau_{t+1}; y_{i,t+1}\right) \tag{12}$$

In choosing which education alternative to provide to its child, the outcome of the political process, (E_{t+1}, τ_{t+1}), is treated as given and does not enter the choices of agents who are atomistic and cannot strategically influence the political process. Given the outcomes of each educational choice represented in equations (10) and (12), the agent chooses the education system for their child that maximises their own utility.

The equilibrium is further characterised by determining the income of an agent who is indifferent between public and private education. This is obtained by equating (10) and (12) for some given period $t + 1$, and public education outcome, $\left(E^*_{t+1}, \tau^*_{t+1}\right)$:

$$V^r\left(\tau^*_{t+1}; y_{i,t+1}\right) = V^u\left(E^*_{t+1}, \tau^*_{t+1}; y_{i,t+1}\right) \tag{13}$$

and solving for $y_{i,t+1} = y^*_{t+1}$ that makes an agent indifferent between education systems.

Proposition 1 *For a given public education combination, $\left(E^*_{t+1}, \tau^*_{t+1}\right)$, the income at which agents will be indifferent between public and private education is given by:*

$$y^*_{t+1}\left(E^*_{t+1}, \tau^*_{t+1}\right) = \frac{E^*_{t+1}(1 + a)^{\frac{1+a}{a}}}{\left(1 - \tau^*_{t+1}\right) a} \tag{14}$$

(i) *This income level, $y^*_{t+1}\left(E^*_{t+1}, \tau^*_{t+1}\right)$, will be unique for any given public education combination, $\left(E^*_{t+1}, \tau^*_{t+1}\right)$.*

(ii) *All agents with income* $y_{i,t+1} \leq y^*_{t+1}\left(E^*_{t+1}, \tau^*_{t+1}\right)$ *will prefer to provide public education for their child.*

This proposition is proved in the appendix.

The result means that any generation can be divided into private and public school consumers for any given public education combination by looking at incomes. Increasing the weight on education, a, means that parents care more about the education of their children. This will reduce the income threshold, y^*_{t+1}, and more parents will opt out of the public system in favor of private education.[4]

The result also provides the proportion of agents consuming public education for a given public education combination, $\left(E^*_{t+1}, \tau^*_{t+1}\right)$, given by:

$$P_{t+1}\left(E^*_{t+1}, \tau^*_{t+1}\right) = \int_0^{y^*_{t+1}} f_{t+1}(y)\, dy = F_{t+1}\left(y^*_{t+1}\right) \tag{15}$$

where $F_{t+1}(\cdot)$ is the cdf and $f_{t+1}(\cdot)$ is the pdf of human capital in period $t+1$.

4 Voting Problem

The equilibrium public education choice, $\left(E^*_{t+1}, \tau^*_{t+1}\right)$, is determined through a voting process. The voter's problem is outlined below and a voting equilibrium is characterised using single crossing monotonicity conditions and results from Epple and Romano (1996).

Each agent i, born in period t, votes for a public education combination that solves the following problem:

$$\max_{E_{t+1}, \tau_{t+1}} V\left(E_{t+1}, \tau_{t+1}; y_{i,t+1}\right) \tag{16}$$

subject to

$$E_{t+1} = \frac{\tau_{t+1} H_{t+1}}{P_{t+1}\left(E_{t+1}, \tau_{t+1}\right)} \tag{17}$$

where income $(y_{i,t+1})$ is known and $V(\cdot)$ is the indirect utility of an agent once an education system has been chosen, given by:

$$V\left(E_{t+1}, \tau_{t+1}; y_{i,t+1}\right) = \max\left\{V^r\left(\tau_{t+1}; y_{i,t+1}\right), V^u\left(E_{t+1}, \tau_{t+1}; y_{i,t+1}\right)\right\} \tag{18}$$

where V^r and V^u are defined in (10) and (12).

[4] This is for a given equilibrium public education combination, $\left(E^*_{t+1}, \tau^*_{t+1}\right)$. A change in a will also change this equilibrium combination and the overall effect on enrollments is indeterminate.

All variables in equations (16) to (18) which comprise the voting problem are dated in the same period. The implication is that even though the model is dynamic, the voting problem of each old cohort is self contained and can be treated as a static voting problem; all time subscripts are omitted from the analysis of the voting problem without loss of generality. This result is due to the altruism assumption that parents are concerned only about the education of their child rather than their child's utility.[5]

Conditions for the existence of an equilibrium in the voting problem outlined above are provided in Epple and Romano (1996), who assume general utility specifications. The problem is also tackled in Glomm and Ravikumar (1998) where CES preferences are assumed. The approach here is to identify the relevant results from Epple and Romano (1996) and ensure that these results will hold in this case.

The one critical characteristic of the voting problem in identifying an equilibrium is the slope of indifference curves in (E, τ) space which is represented by $M(E, \tau; y_i)$. This is the marginal rate of substitution between taxes and public education expenditures, which is found by differentiating equations (10) and (12):

$$M(E, \tau; y_i) = \begin{cases} \frac{a(1-\tau)}{E} > 0 & \text{if } y_i \leq y^*(E, \tau) \\ 0 & \text{if } y_i > y^*(E, \tau) \end{cases} \tag{19}$$

When $y_i > y^*(E, \tau)$, the agent prefers private education and for a given tax rate, increases in E will not change utility, hence indifference curves in (E, τ) space are horizontal and $M(\cdot) = 0$. Agents that have $y_i \leq y^*(E, \tau)$ prefer public education, with positively sloped indifference curves whose slope do not depend on income, y_i. Increases in the tax rate must be accompanied by increases in public education expenditure to maintain constant utility. These indifference curves are illustrated in Figure 1.

Result 1 (Epple and Romano (1996), Proposition 1) *When preferences are such that $\partial M / \partial y_i \leq 0$, a majority voting equilibrium exists, and the median income voter is decisive.*

Inspection of (19) verifies $\partial M / \partial y_i = 0$ and that Result 1 will be applicable to the present problem. The Epple and Romano result imposes a single crossing condition. The intuition is that if we consider the median income voter's most preferred voting outcome, all those

[5] The latter alternative assumption would have the parent concerned about all future generations utility and significantly complicate the analysis.

with incomes below median income will prefer higher or the same taxes and all those with incomes above median income will prefer lower taxes, as long as $\partial M / \partial y_i \leq 0$. This is instead of requiring the stricter condition of preferences being single peaked over taxes. In the present case, $\partial M / \partial y_i = 0$ means that all agents that prefer public education will vote for exactly the same non-zero tax rate as the median income voter, while all agents that prefer private education will vote for a zero tax rate, $\tau = 0$. The political outcome comprises two homogeneous voting coalitions. The following assumptions ensure that the model does not collapse into a universal public or universal private education model.

Assumption 1 *The median income voter prefers public education, $e_i = 0$, to private education, $e_i > 0$.*

Assumption 2 *The highest income voter prefers private education, $e_i > 0$, to any equilibrium level of public education.*

Both of these assumptions ensure a non-trivial education choice model and are related to the distribution of income. Assumption 1 requires that median income is less than mean income; that the distribution of income is positively skewed.[6] The assumption ensures that public education expenditure will be positive in equilibrium. Assumption 2 ensures that at least one agent opts out of the public education system and requires the income distribution to be sufficiently positively skewed.

5 Dynamics

The evolution of a given family's human capital and income depends on the choice of education system across generations. Whether an agent has received private or public education will determine the dynamic behavior of income. This can be formally illustrated by substituting the equilibrium private and public education expenditures, from (9) and (17), into (7). The dynamic equation for income and human capital when private education is provided by the parent is:

$$h^r_{i,t+1} = \theta h^\gamma_{i,t} e^{1-\gamma}_{i,t} = \theta \left(\frac{a\,(1-\tau_t)}{(1+a)} \right)^{1-\gamma} h_{i,t} = A_t h_{i,t} \tag{20}$$

[6] See Epple and Romano (1996a) for details on how Assumption 1 implies positive skewness of the income distribution.

This linear relationship between $h^r_{i,t+1}$ and $h_{i,t}$ applies to all agents that received private education when young, those whose parents income $y_{i,t} > y^*_t$ as defined in Proposition 1. These agents will experience linear growth dynamics or endogenous growth as long as $A_t = \theta\left(\frac{a(1-\tau_t)}{(1+a)}\right)^{1-\gamma} > 1$.

Alternatively, when public education is provided by the parent, the dynamic equation for income and human capital is given by:

$$h^u_{i,t+1} = \theta h^\gamma_{i,t} E^{1-\gamma}_t = \theta\left(\frac{\tau_t H_t}{P_t}\right)^{1-\gamma} h^\gamma_{i,t} \tag{21}$$

This non-linear relationship applies to all public education students, those whose parents income $y_{i,t} \le y^*_t$, and resembles neoclassical growth dynamics. The dynamic behavior of income and human capital is illustrated in Figure 2. The human capital of families that use public education will converge to a fixed point given by:

$$h^s_t = \theta^{\frac{1}{1-\gamma}}\left(\frac{\tau_t H_t}{P_t}\right) \tag{22}$$

which resembles a steady state, except for its dependance on economy wide, time varying variables such as τ_t, H_t and P_t. Households with human capital, $h_{i,t}$, above (below) h^s_t will move down (up) towards this fixed point in period $t+1$. This fixed point can however vary over time and between generations, depending on how τ_t, H_t and P_t evolve.

Unlike a typical neoclassical growth model, public education students evolving according to equation (21) can experience long run growth, through the dependance on a growing average human capital, H_t.[7] While private education students experience growth described by (20), the average human capital stock grows, raising h^s_t over time. Thus public education students are experiencing a spillover which is illustrated in Figure 3. The intuitive mechanism is that income and thus tax base growth due to private education raises the public education budget leading to a fiscal spillover and long run growth in the incomes of public education students.

However, this mechanism also depends on P_t and τ_t which can both vary over time. Growth in H_t raises E_t, making public education more attractive and acting to raise P_t, but it also motivates reductions in the tax rate τ_t, a substitution effect, thereby acting to

[7]The classical example of this dependence on average human capital is Lucas (1988).

reduce P_t. The net effects are complex and cannot be analytically determined. Instead, the behaviour of public and private education students are studied using simulations below.

One interesting characterisation of growth in the mixed education model can however be made. Figure **??** illustrates the dynamic behaviour of income and human capital for families under the two alternative education regimes. In particular, it shows the possibility that growth for an individual family can be higher under the public education alternative than under the private education alternative. This is the case for those whose parents have very low human capital, identified in the following proposition.

Proposition 2 *For a given public education combination, $\left(E^*_{t+1}, \tau^*_{t+1}\right)$, all families with human capital below the threshold $\widehat{h}_{t+1}$ will attain higher human capital in the next generation under public education than under private education, where $\widehat{h}_{t+1}$ is given by:*

$$\widehat{y}_{t+1} = \widehat{h}_{t+1} = \frac{E^*_{t+1}\left(1+a\right)}{a\left(1-\tau^*_{t+1}\right)} \tag{23}$$

Proposition 2 is proved in the Appendix of the paper.

The intuition of this result is that these low income families receive much greater public education expenditures than they could afford to make themselves in the private education alternative system. Thus the higher public investment in human capital drives their faster growth. The growth threshold can be compared to the indifference threshold from Proposition 1.

Corollary 1 *The threshold income for indifference between public and private education, y^*_{t+1} given in Proposition 1 always exceeds the threshold income for public education to provide higher growth than private education, $\widehat{y}_{t+1}$ given in Proposition 2:*

$$y^*_{t+1} > \widehat{y}_{t+1} \tag{24}$$

Corollary 1 is proved in the Appendix of the paper.

The corollary indicates that not all of the families using public education will experience the higher growth identified in Proposition 2. There are a group of public education families, with incomes between y^*_{t+1} and $\widehat{y}_{t+1}$, that would experience higher growth if they could afford private education. These analytical results will be used below to analyse the results of simulations.

A closed form for the distribution of income cannot be determined, primarily because of the bifurcation in the growth dynamics. It can however be seen that the income distribution will separate into two distinct groups. The highest income public education family experiences decay, converging to h_t^s, while the lowest income private education family experiences growth, moving away from the wealthiest public education family, and h_t^s, thus causing the income distribution to separate into two groups.

Equation (20) shows that for private education students, income depends on two heterogeneous factors, education spending and parent's human capital. The linear relationship found in equation (20) implies that relative income inequality among private education students will not change over time. The size of the parameter γ will not affect the evolution of income inequality among private education students, but (20) shows that, given other parameter values, γ must be sufficiently small for $A_t = \theta\left(\frac{a(1-\tau_t)}{(1+a)}\right)^{1-\gamma} > 1$, ensuring that private education students will experience growth rather than decay or a multiplicity of equilibria. The larger is γ, the smaller A_t will be and the slower the growth experienced by private education students. This has implications for public education enrollments. As per capita income grows, public education improves and the threshold of indifference, $y_{t+1}^* = h_{t+1}^*$, rises. For larger values of γ, private education families grow more slowly and are more likely to find their income overtaken by the threshold $y_{t+1}^* = h_{t+1}^*$ at some point in time. Thus, lower values of γ lead to more families dropping out of private education because they cannot afford it, resulting in greater public education enrollments, this is studied using simulations below.

For public education students, equation (21) shows the only source of heterogeneity in the accumulation of human capital across generations is parent's human capital; education expenditure is identical across all public education students. The effect of this heterogeneity is determined by the parameter γ. Public education human capital and incomes converge to h_t^s more quickly with smaller values of γ. The intuition is that a smaller value of γ puts more weight on education expenditure, which is homogeneous, and less weight on parent's income, which is heterogeneous, in the accumulation of human capital. The result is that the convergence to the public education fixed point, h_t^s, is faster with smaller values of γ.

5.1 A Special Case: $A_t = 1$

Under the assumption that $A_t = 1$, a steady state exists and the behaviour of the income distribution can be characterised. The incomes of the private education families follow (20) with $A_t = 1$ substituted and are thus stationary. This means that all private education families are in their steady states from the first period.

The incomes of the public education families in this case converge to the fixed point defined by equation (22). Per capita income evolves over time, changing as the incomes of public education household's incomes move closer to the fixed point given by (22).[8] During this evolution, public education may improve and taxes may change, there is a possibility that some private education families may switch to public education. In the long run however, all incomes are stationary, public education incomes are all identical while the distribution of private education incomes follows the upper tail of the initial distribution of income.

There will be an income gap between the public education steady state and the lowest private education income, see Figure 2. The public education steady state will be higher than in a pure public education model (discussed below) if public education expenditure, $E_t = \frac{\tau_t H_t}{P_t}$, is higher in the mixed model. This would be expected as $P_t < 1$ and per capita income, H_t, would both be higher in the mixed education model, however the size of the tax will play a role but cannot be explicitly determined, thus precluding any analytical determination.

Extending the analysis of this special case to the case where $A_t > 1$, incomes and thus the distribution are not stationary but the distribution behaves in a very similar manner. All public education incomes converge to the fixed point defined by equation (22), though the value of h_t^s changes over time, and all private education incomes grow at identical constant rates, as implied by (20), and exhibit the same relative inequality as in the initial distribution of income. Thus we expect all incomes to exhibit long run growth in the general case but the distribution will resemble that described for the case where $A_t = 1$. The general case is investigated further using simulations below.

[8] Private education incomes do not change and thus do not drive changes in per capita income.

5.2 Benchmark Models

The mixed education model outlined above is to be studied using numerical simulations below and is to be compared to universal public and universal private education benchmarks. These benchmark models can be derived from equations (1) to (6), by setting $e_{i,t} = 0$, $\forall i$ and $P_t = 1$ to derive the pure public education model and setting $E_t = P_t = 0$ to derive the pure private education model. The equilibria in each of these models can be derived as in Sections 3 and 4. The dynamics of these models are as follows.

In the purely private education version of the model, human capital evolves according to the following dynamic equation:

$$h_{i,t+1}^{rb} = \theta h_{i,t}^{\gamma} e_{i,t}^{1-\gamma} = \theta \left(\frac{a}{(1+a)} \right)^{1-\gamma} h_{i,t} \tag{25}$$

which is similar to that presented in (20). The main difference is that in a pure private education model, there are no taxes and thus the slope of the linear growth path is greater. Since all families follow this growth dynamic, relative inequality remains unchanged.

In the pure public education system, human capital evolves according to:

$$h_{i,t+1}^{ub} = \theta h_{i,t}^{\gamma} E_t^{1-\gamma} = \theta \left(\tau_t H_t\right)^{1-\gamma} h_{i,t}^{\gamma} \tag{26}$$

which is in turn similar to the dynamic equation presented in (21). The human capital of the whole population will converge to a fixed point that is derived in a similar way to and resembles equation (22). An important point to note is that eventually the population will become completely homogeneous, resembling a representative agent. Once this occurs, $H_t = h_{i,t}$, $\forall i$, while the equilibrium tax rate is $\tau_t = \frac{a}{(1+a)}$, so the dynamics will revert to exactly those of the private education model given by (25). The key difference is that in this long run public education equilibrium, the income distribution is degenerate, all agents are identical, while in the long run private education equilibrium the income inequality observed in the first generation remains. These two economies will grow at the same rate but because of the heterogeneity in the private education model, per capita income, average education expenditure and of course income inequality will differ between the two models.

6 Simulations

Simulations are used to further analyze the dynamic behavior of the model and to illustrate the endogenous distribution of income. The approach is to simulate the model for a large random sample of agents and use the income distribution of the sample as an approximation of the actual endogenous income distribution. Average incomes and education expenditures are plotted over time and compared to benchmark cases of universal public and private education.

The structure required to simulate the model involves first assuming some distribution for income of the first generation. The income distribution is chosen to be lognormal with a mean of \$47101 and a median of \$35172 in order to match US household incomes in 1996, as reported by McNeil (1998).[9] The second issue is model parameters. A wide range of model parameter combinations are possible, empirical evidence suggests that γ, which measures heritability of human capital and income, be relatively large,[10] while other recent simulation work using similar human capital production functions suggests $(1-\gamma)$ is in the range of 0.05 to 0.2.[11] Thus a value of $\gamma = 0.9$ is chosen while $\gamma = 0.8$ and 0.7 are also considered in sensitivity analyses. It is also important that θ be chosen large enough to ensure growth rather than decay for the private education consumers,[12] $\theta = 1.6$ is used. The preference parameter on education, a, is chosen to match per student education spending in US public schools in 1994-95 which is reported to be \$5494 in the US Bureau of the Census, Public Elementary-Secondary Finances: 1994-95. A value of $a = 0.13$ matches the first generation's public education budget to this data. The sample size used to approximate the distribution of income is chosen to be 10000. Details of the simulation process are contained in the appendix at the end of the paper.

[9] The lognormal assumption means that $\ln(h_{i,0}) \sim N(\mu, \sigma^2)$, with $\mu = 3.56$ and $\sigma = 0.77$, where $h_{i,0}$ are the human capital endowments of the first generation who are born old. See Aitchison and Brown (1957) for details of these results.

[10] See Aughinbaugh (2000), Solon (1992) and Zimmermann (1992) for evidence on this.

[11] See for example Glomm and Ravikumar (1998b) and Fernandez and Rogerson (1998).

[12] See equation (20) in Section 5.

6.1 Results

Figure 4 provides a comparison of the education choice model with the benchmark private and public education models, with parameter values $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.13$. This parameterisation is referred to as the baseline case. The figure makes comparisons of average incomes, education expenditures and income inequality, measured with Gini coefficients, in the three models of education.

Panel (a) of Figure 4 is a plot of the ratios of public to private per capita income (dashed line) and mixed to private per capita income (solid line). This illustrates that average incomes in the mixed education model are higher than in the public education model. The ratio of mixed education income to private education income starts off at one and declines over time, indicating that the mixed education model provides per capita income below that of pure private education. The results indicate that the per capita income in the mixed education model will lie between per capita incomes in the benchmark models. The main reason for this is the fiscal spillover discussed in Section 5 and illustrated in Figure **??**. Public education students, along with private education students, experience long run growth because of this spillover, in turn keeping the per capita income of the economy above that of a pure public education economy.

Inferences about growth in the mixed education model can be made from panel (a) of Figure 4. The slope of these two curves tells us the rate of growth relative to the private education benchmark model. When the curve is horizontal, zero slope, the model in question is growing at the same rate as the private benchmark. A positive(negative) slope implies growth greater(less) than in the private education benchmark. Growth in both the public benchmark and the mixed education model is lower than in the pure private education benchmark. After 20 periods, the public education benchmark converges to its steady state and the rate of growth is the same as in the private benchmark (implied by the horizontal part of the dashed curve in panel (a)), consistent with analysis in Section 5.2. The mixed education model offers greater growth than the public education benchmark in the first 10 periods but once the public education benchmark has converged, the mixed education model generally offers slower growth than the pure public education model. One reason for this result is that in the mixed education model, there is a flow of students out of private education into public; see Table 1 for details of this. These switchers will experience a drop in income until they reach the public education fixed point, causing a lower growth rate.

Panel (b) of Figure 4 is a plot of the ratios of average public to average private education expenditure (dashed line) and average mixed to average private education expenditure (solid line), while the solid horizontal line represents education expenditure in the private education model. The mixed education model initially provides greater average and total education expenditure than both the private and public education models. This occurs because public education ensures expenditure on those below mean income is greater than they would make on their own in private education (Proposition 2). Given the positive skewness of the income distribution, this relates to the majority of the population. On the other hand, those choosing private education can spend more on education than they would receive in public education. Taken together, education expenditure in the mixed education model exceeds that in either of the benchmarks in the first few periods.

After 4 periods, mixed education model education expenditure falls below that in the private education benchmark, primarily because of the numbers of families switching from private to public education. As per capita income grows, public education improves and becomes more attractive to the poorest private education families and can be seen in Table 1. This is why over time families are dropping out of the private education system and into the public education system. When this happens the tax rate must increase in order to maintain the quality of public education. The jumps in the solid line in panel (b) are the result of changes in the equilibrium tax rate in the mixed education model due to such drop outs. The discrete jumps are unexpected and are the result of the simulation technique which involves a tax grid. Taxes cannot rise infinitesimally but must move up to the next value on the grid. If a finer grid were implemented, the jumps would be smaller and the behaviour of education spending would be smoother.[13] For a given tax rate, an increase in public education enrollments will reduce the quality of public education and induce increases in taxes, in order to maintain public education quality. This is illustrated in Table 1 which presents tax rates and public education enrollments. As public enrollments rise, so too do taxes. When public enrollments are stable, tax rates fall, because of the growth in per-capita

[13] A grid of 1000 tax points is employed, increments of 0.001. Increasing the fineness of this grid would be computationally expensive and would probably not qualitatively alter the results.

income, driven primarily by a spillover from growth in private education income.

Panel (c) of Figure 4 plots the Gini coefficients for the Public (short dashed line), Private (dashed line) and mixed education model (solid line). The public education benchmark model exhibits income convergence with the Gini coefficient falling over time. The private education benchmark model exhibits constant inequality, all incomes grow at the same rate hence the initial levels of relative inequality are maintained. Of primary interest is income inequality in the education choice model, which lies between the private and public education inequality levels, declines over time and approaches the benchmark public education values. Two opposing forces are at work on the distribution. First is the income convergence experienced by public education students, acting to reduce income inequality, as discussed in Section 5. Second is the income gap between the public and private education students, also discussed in Section 5. Despite the emergence of an income gap, income inequality, measured by the Gini coefficient does approach zero. The results suggest that over time, a mixed education model can offer reduced income inequality without reducing per capita income to the extent that public education does.

Income distributions for the education choice model are illustrated in Figure 5. Panel (a) of Figure 5 is the initial lognormal distribution described above. Panel (b) of Figure 5 shows the income distribution after one period, panel (c) after 10 periods, panel (d) after 20 periods, panel (e) after 30 periods and panel (f) after 40 periods. One of the main points of these distributions is to show that two distinct groups form endogenously and there is an income gap between the two groups. This result is clearest in panels (b), (c) and (d). Members of the lower income group all consume public education and the upper income group all consume private education. The upper income group shrinks over time and is relatively small and difficult to discern in panels (e) and (f) after 30 generations, which suggests another important point, that models of mixed provision may exhibit some dynamic instability. In this case, growth in public education enrollments has required the tax rate to rise from 9.6% in the first period to 10.6% after 40 periods; see Table 1. This increasing tax burden forces the marginal private consumers out of private education and in turn requires further increases in taxes as is evident again from Table 1. In the long run, only the extremely wealthy use private schools.

6.2 Sensitivity

In this section, the sensitivity of the above results are tested with respect to changes in γ, a and the initial income distribution. One of the more important parameters in the simulations is γ, the relative importance of education in the accumulation of human capital. As discussed in Section 5, larger values of γ will slow the growth of private education students and increase the public eduction fixed point, h_t^s, given by (22). This makes public education more attractive relative to private education, leading more students to switch from private to public education. Larger values of γ also lead to faster convergence within the public education group, and lower levels of overall inequality.

Simulations are carried out with $\gamma = 0.8$ and 0.7, with analogues of Figure 4 presented in Figures 6 and 7. These figures confirm the analytical predictions above. As γ decreases, families leave the private education system earlier in the simulations, relative to the baseline case. In Figure 7, for the case where $\gamma = 0.7$, the private system shuts down with no private education students at all by the end of the 60 generation simulation. This is indicated by the horizontal portion of the income ratio curve for the mixed education model in panel (a) of Figure 7. It means that all families have left the private education system and the model has become a pure public system, growing at the same rate as the benchmark public education model.[14] Comparing Tables 2 and 3 with Table 1 also confirms that the smaller is γ, the faster the exit from and closure of the private education system will be. The analogues of the distributions in Figure 5 for the cases where $\gamma = 0.8$ and 0.7 bear out the fact that private education enrollments are smaller at any given time for smaller values of γ. However, these figures are not qualitatively different from Figure 5 and are thus not presented.[15]

The next test of sensitivity involves varying a, the weight parents place on education in their utility. The values considered are half and double the baseline value of $a = 0.13$. Simulations using the baseline parameterisation but with $a = 0.065$ and 0.26 are carried out, with the analogue of Figure 4 presented in Figures 8 and 9. Panel (a) of Figure 8 shows that when $a = 0.065$, the mixed education model grows faster than the public and

[14] The mixed model curve is slightly lower than the public education benchmark because of the approximation implemented by the tax grid in the mixed education model, while exact solutions are calculated for the benchmark models.

[15] These Figures have been included in an Appendix as Figures A1 and A2 for the benefit of the referees and will be available to readers upon request unless the editor or referee suggest their inclusion.

at times even the private education benchmark model. The reason for this can be seen by comparing Table 4 with Table 1. The lower weight on education means that more people can afford and are willing to opt out of the public education system, primarily because the tax rate required to finance the public education system is lower. The larger private education sector leads to a larger fiscal spillover, in turn leading to higher incomes and faster growth relative to the public education benchmark, while the mixed education model almost matches the incomes of the private eduction benchmark. Looking at panel (b) of Figure 8 shows that average education spending in the mixed education model almost matches that in the private education benchmark, more confirmation of a larger fiscal spillover. Finally, panel (c) of Figure 8 confirms that inequality in the mixed education model is higher, compared to the baseline case in Figure 1, when $a = 0.065$, as would be expected from the results in Table 4.

The other extreme, where $a = 0.26$ produces the opposite results, see Figure 9. Education is more important to all families and as a result, a higher tax is required to finance the equilibrium public education system. This in turn forces more families out of the private education system at earlier stages. Both the higher tax rate and the earlier exit are evident in Table 5. The fiscal spillover is smaller because of lower private education enrollments, leading to lower per capita income and lower average education expenditures relative to the baseline case of $a = 0.13$, see panels (a) and (b) of Figure 9. Table 5 indicates the lower private education enrollments and suggests lower inequality. This is confirmed by panel (c) of Figure 9 which indicates lower inequality in the mixed education model when compared to the baseline case of $a = 0.13$, panel (c) of Figure 4.

For both these cases, the analogues of the distributions in Figure 5 do confirm the higher (lower) private education enrollments for $a = 0.065\,(0.26)$. These figures show no major qualitative difference from Figure 5 and are not presented.[16]

The sensitivity of the results to the assumed income distribution is studied by considering a distribution with the same median, \$35172, and twice the mean, \$94202, thereby increasing initial inequality and positive skewness. Figure 10 presents comparisons of the mixed education model with the public and private education benchmarks and is analogous to Figure

[16] Once again, these Figures have been included in an Appendix as Figures A3 and A4 for the benefit of the referees and will be available to readers upon request unless the editor or referee suggest their inclusion.

4. The results are quite different from those in the baseline case. Per capita income in the mixed education model is higher than in both the public and private education benchmarks, panel (a) of Figure 10. This is driven by the higher education expenditures in the mixed education model than in either of the benchmark models. The source of the higher education spending is the larger number of private education families, see Table 6, and the fact that they are wealthier (due to the increased skewness). The fiscal spillover discussed above has been increased by the change in the distribution. The result suggests that inequality may be beneficial to growth and has implications for education policy in economies where income distribution may resemble the increased skewness, possibly developing economies. Panel (c) of Figure 10 indicates the higher income inequality in the mixed education model compared to that in panel (c) of Figure 4, this is expected as initial inequality has been increased and is in line with results in Table 6. In this case, the distributions of income are again, not qualitatively different from that presented in Figure 5 and are again not presented.[17]

7 Conclusions

This paper has studied the dynamic evolution of an economy in which parents chose to send their child to either a public or private school, while the tax rate that supported public schools was determined by a majority vote of all parents. The key innovation of the paper was the combination of an education choice framework with growth dynamics. This enabled the study of income distribution and growth over time in a model of education choice where both public and private education alternatives coexisted. This raised political tensions between public and private education voters in a dynamic setting and led to interesting interactions between the groups in the form of growth spillovers.

Equilibria under the alternative education systems were characterized and an endogenous income threshold that was used to sort students into public and private schools was identified. The voting problem was also characterized and each old cohort's voting problem was found to be self-contained and independent of other generation's decisions. This enabled the identification of a voting equilibrium through the application of results from the work of

[17] These figures have also been included in an Appendix as Figure A5 for the benefit of the referees and will be available to readers upon request unless the editor or referee suggest their inclusion.

Epple and Romano (1996). Those with low incomes preferred public education and higher tax rates while those with high incomes preferred private education and lower tax rates.

The dynamic behavior of the incomes of public and private education students was analyzed. Both public and private education students experienced endogenous growth though through different mechanisms. Private education students experienced endogenous growth through the more typical linear AK dynamics, while public education students experienced endogenous growth via a spillover from the private education group. An endogenous separation in the income distribution based on education was also identified. It was also found that the very poor experience higher growth under the public education alternative because public education is *better* than what could be purchased in the private education alternative. Simulations were used to characterize the endogenous income distribution.

These simulations identified a bimodal income distribution, identifying a class structure based on education. Public education families converged to a low income equilibrium and private education families stayed in a high income equilibrium. The education choice model was compared to benchmark public and private education models and was found to reduce income inequality (relative to the private education benchmark) despite the bimodal income distribution. It was also found that the education choice model provided per-capita incomes greater than those found in the public education benchmark model, but less than those found in the private education benchmark model.

Individual initial conditions were found to be important to the outcome of the model. A poor family would likely never be able to get out of the public education system, or have incomes as high as their private school contemporaries, while a wealthy family could continue to be privately educated, maintaining their relative wealth. Thus the educational institutional setting appeared to provide a poverty trap or at least limit upward mobility. However, the mixed education system benefited public education students by generating a growth spillover. The private education group's endogenous growth raised the tax base and thus the public education budget. Simulation experiments found that increased positive skewness in the initial distribution of income lead to a larger private education group, a larger growth spillover and in the long run, higher income inequality and per-capita incomes; all relative to an identically parameterised education choice model with lower initial income inequality and skewness. Another important issue identified by senstivity analysis was that such mixed education models may be unstable in the long run, with families switching out of the private education alternative over time. In some cases, the private education alternative ceased to exist in the long run.

The model identifies that while there may be problems with a private alternative to public education, in particular the bimodal income distribution, such an institutional setting can raise growth relative to a compulsory public education system, while still reducing income inequality.

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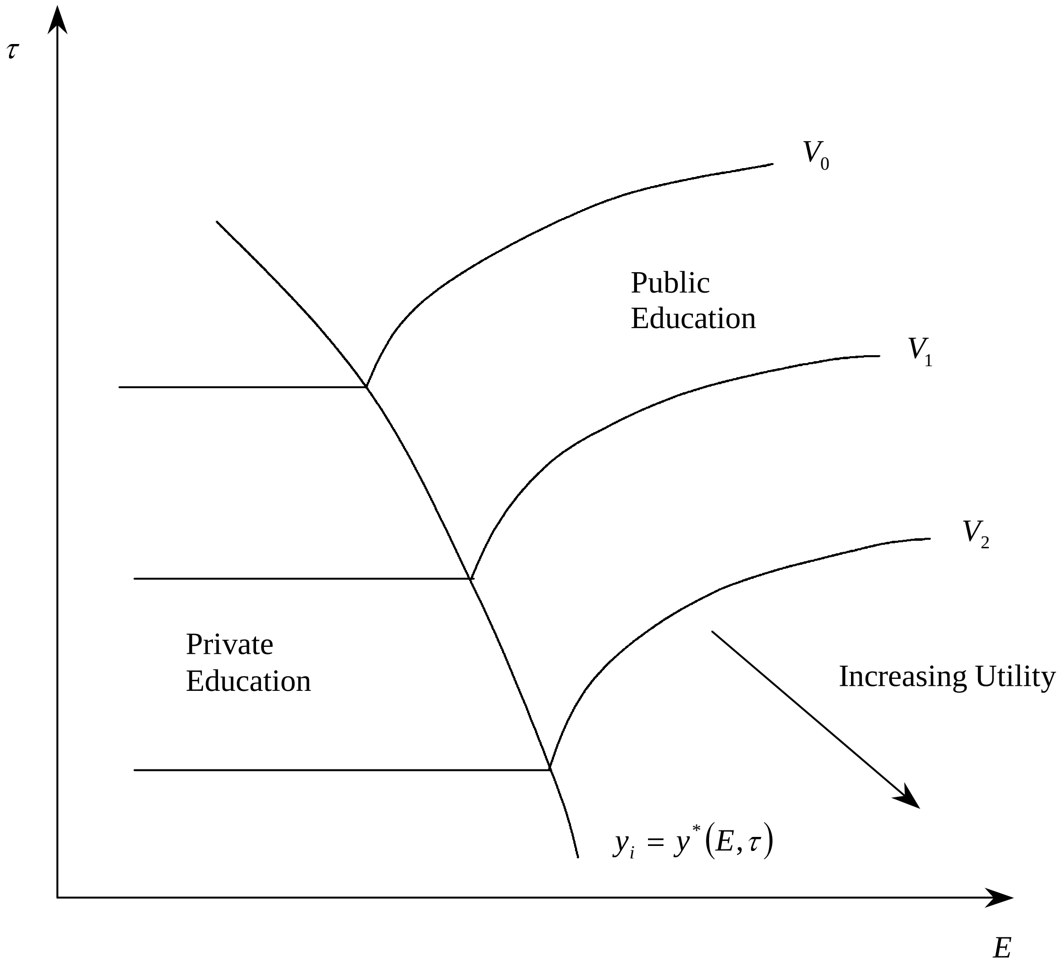

Figure 1: Indifference curves of agent *i*, with income y_i in the (E,τ) plane. Private education is consumed (and preferred) along the horizonal portions and public education along the upward sloping portions. As income (y_i) increases, the locus $y_i = y^*(E,\tau)$ shifts right and private education is preferred for a greater number of (E,τ) combinations.

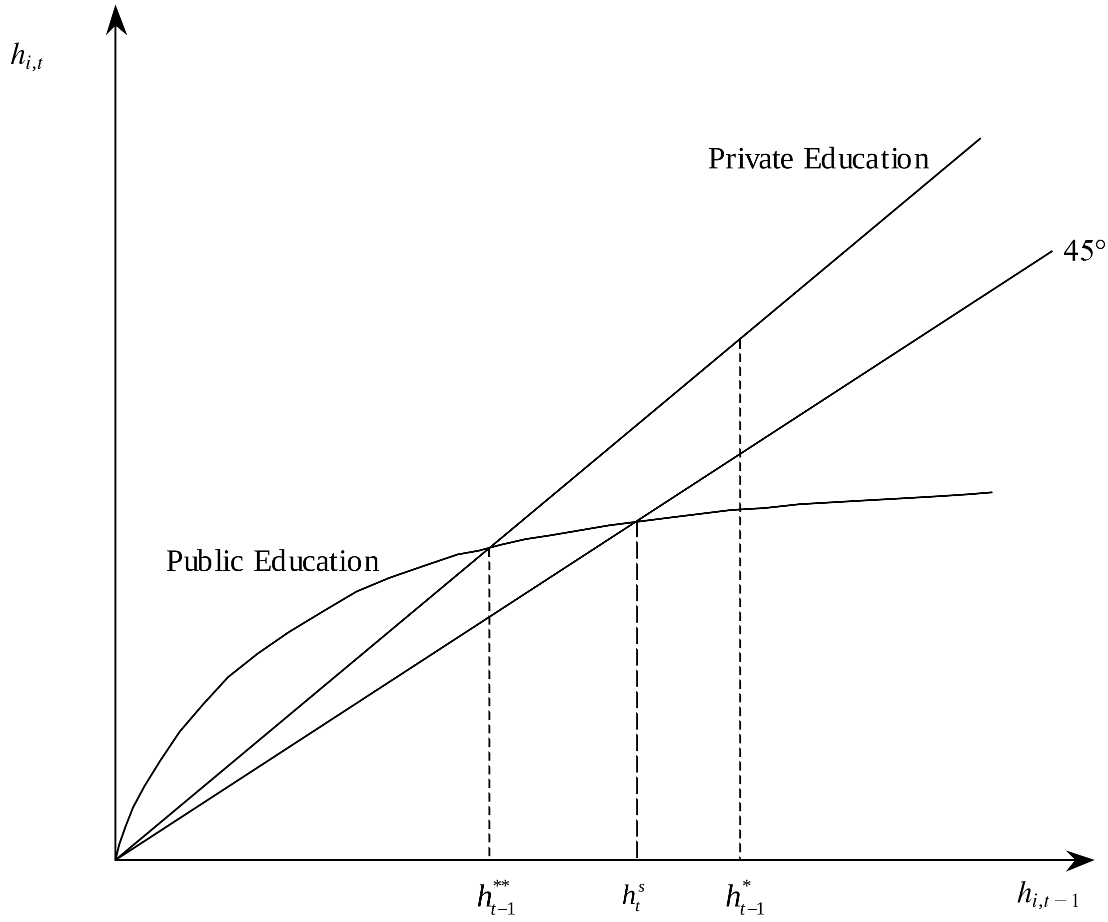

Figure 2: Dynamics for mixed public and private education model. Families with human capital below h_{t-1}^{*} choose public education for their child. Public education offers faster growth for families with human capital below h_{t-1}^{**}. Public education families' human capital converges to the steady state h_t^s while private education families' human capital follows the linear growth path above the 45° line.

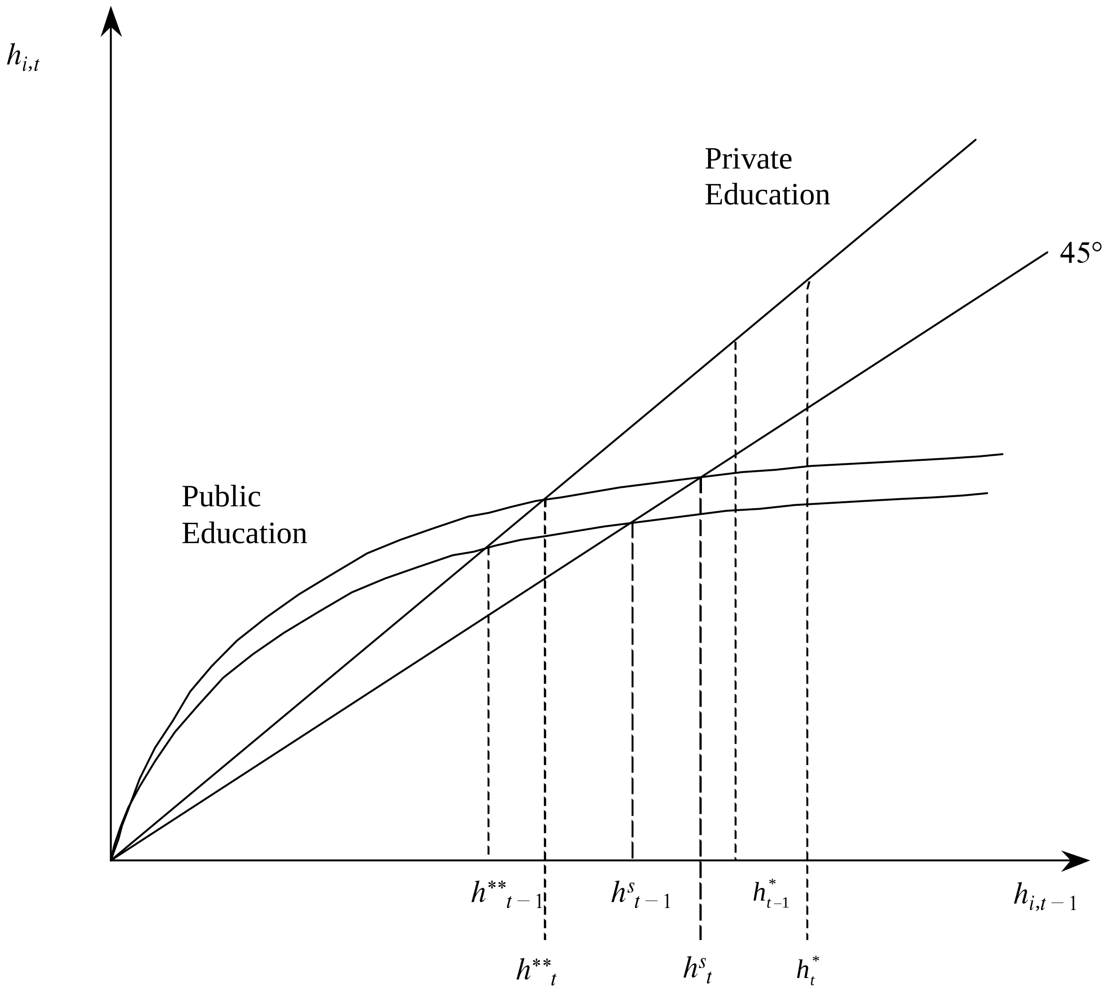

Figure 3: Evolution of dynamics for mixed education model. In period $t-1$, families with human capital below h^{*}_{t-1} choose public education for their child and all public education families converge towards h^{s}_{t-1}. In period t, per capita human capital, H_t has changed and as a result, the threshold for indifference between public and private education has risen to h^{*}_{t} and public education families, those below h^{*}_{t}, converge to the new public education fixed point, h^{s}_{t}.

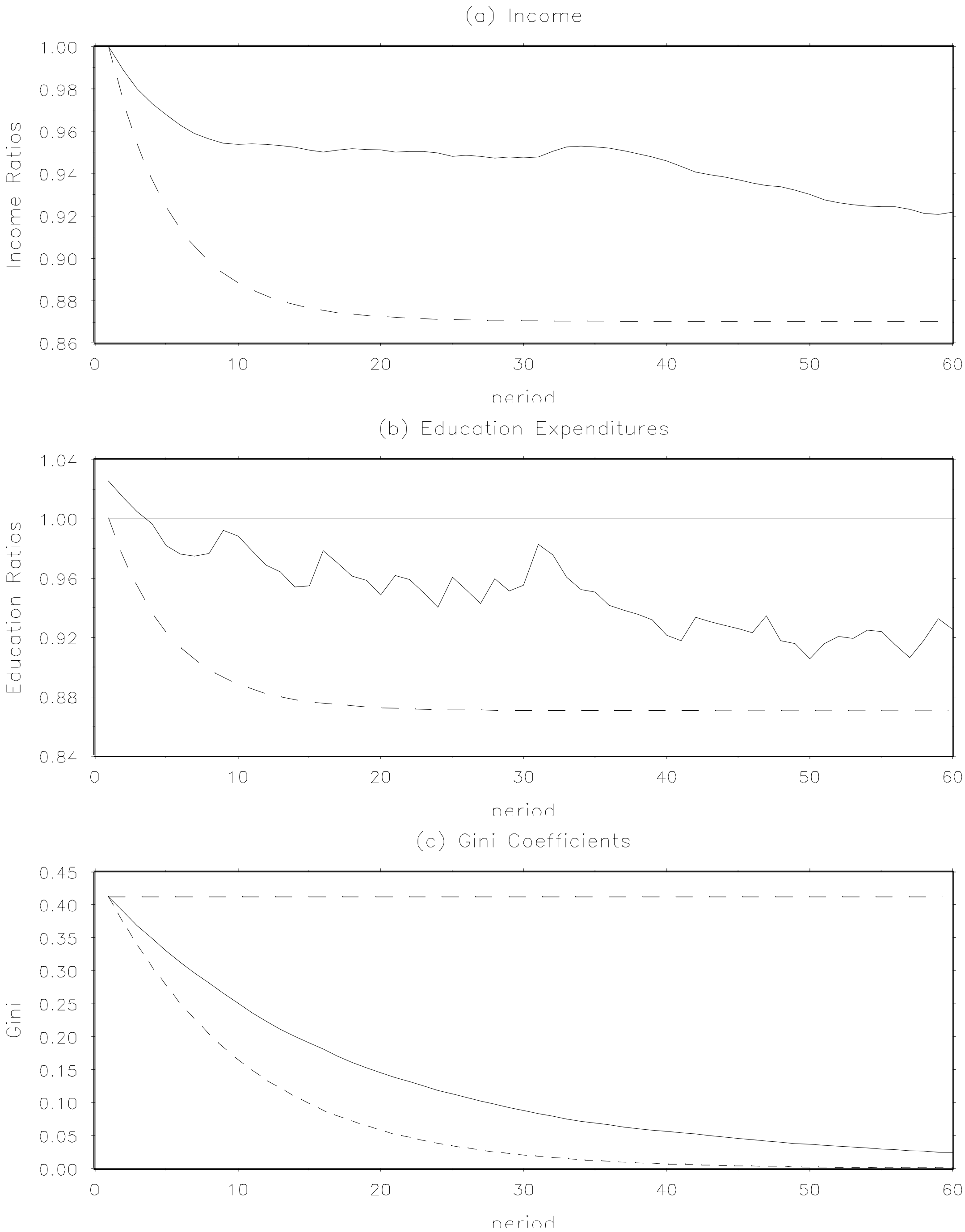

Figure 4: Comparisons of incomes, education expenditures and gini coefficients between the mixed education model and the public and private education benchmark models. In panels (a) and (b), mixed:private model ratios given by solid lines and public:private ratios given by dashed lines with horizontal solid line being private benchmark. The Gini coefficients for the private (dashed line), mixed (solid line) and public (short dashed line) education models are presented in panel (c). Parameter values are $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.13$, with initial income distribution having mean \$47101 and median \$35172.

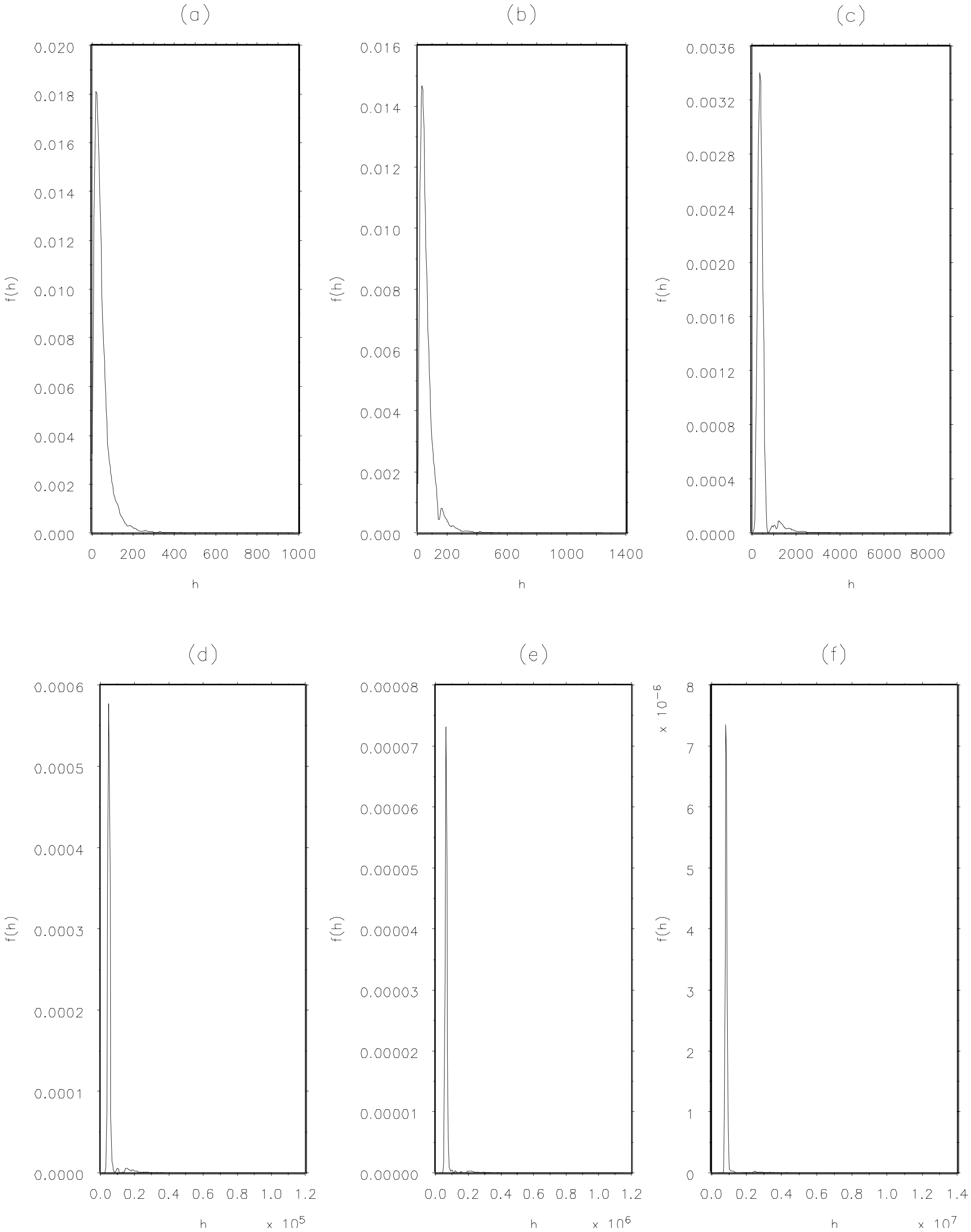

Figure 5: Income Distribution in the model of education choice. Panel (a) provides the initial distribution described above. Panels (b), (c), (d), (e) and (f) provide the endogenous income distributions after 1, 10, 20, 30 and 40 generations respectively, where human capital (income) is measured in thousands of dollars. Parameter values are $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.13$, with initial income distribution having mean \$47101 and median \$35172.

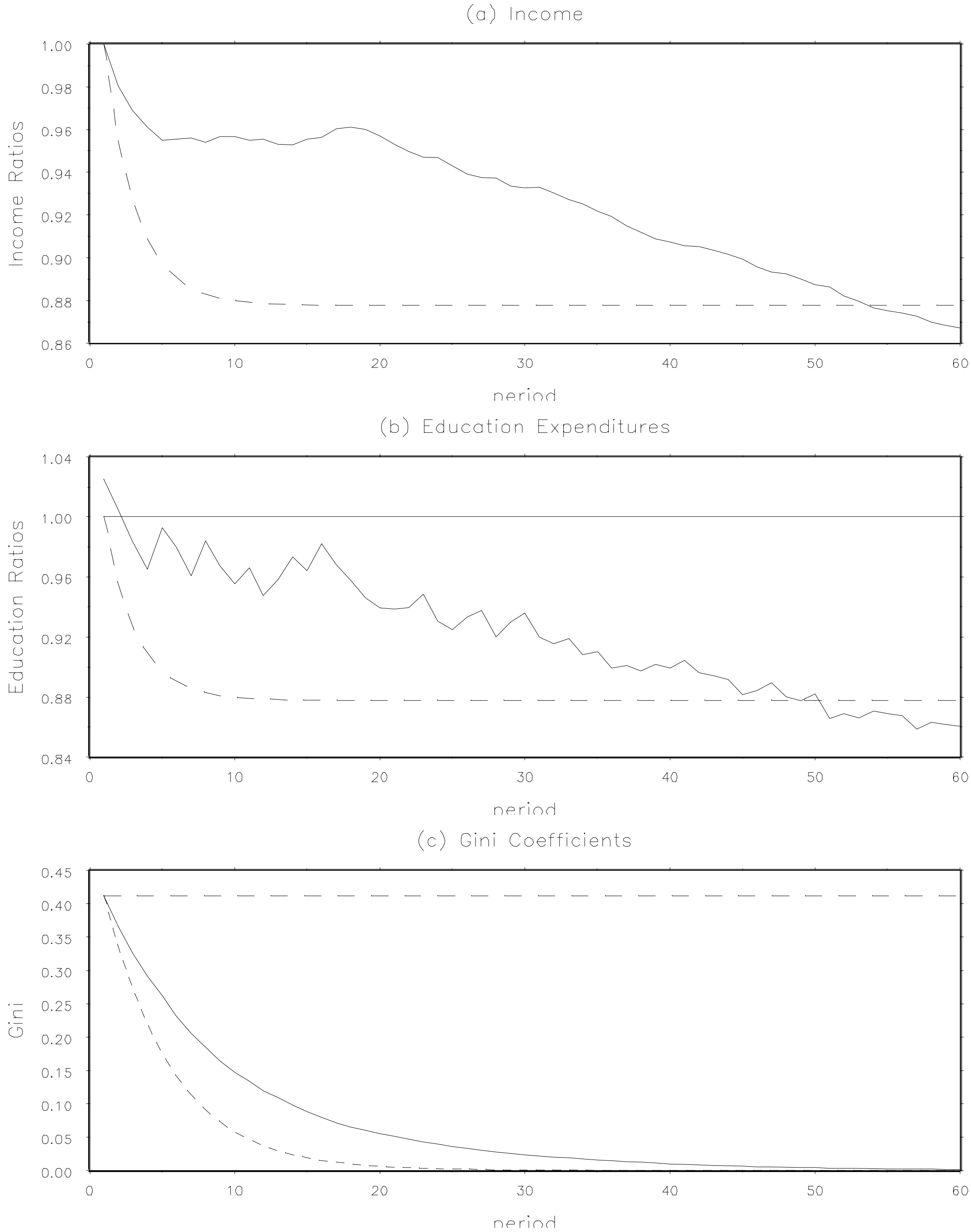

Figure 6: Comparisons of incomes, education expenditures and gini coefficients between the mixed education model and the public and private education benchmark models. In panels (a) and (b), mixed:private model ratios given by solid lines and public:private ratios given by dashed lines with horizontal solid line being private benchmark. The Gini coefficients for the private (dashed line), mixed (solid line) and public (short dashed line) education models are presented in panel (c). Parameter values are $\gamma = 0.8$, $\theta = 1.6$ and $a = 0.13$, with initial income distribution having mean \$47101 and median \$35172.

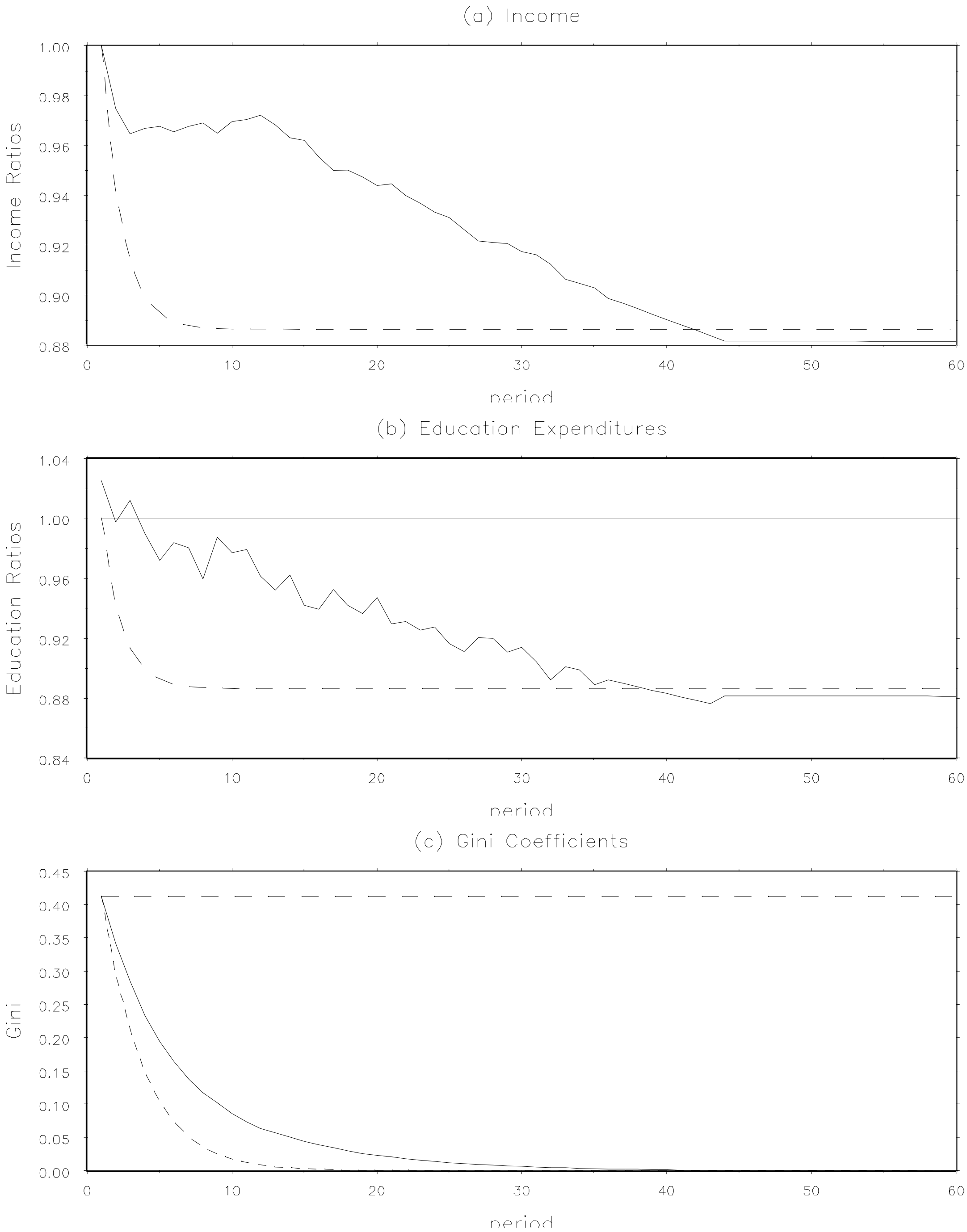

Figure 7: Comparisons of incomes, education expenditures and gini coefficients between the mixed education model and the public and private education benchmark models. In panels (a) and (b), mixed:private model ratios given by solid lines and public:private ratios given by dashed lines with horizontal solid line being private benchmark. The Gini coefficients for the private (dashed line), mixed (solid line) and public (short dashed line) education models are presented in panel (c). Parameter values are $\gamma = 0.7$, $\theta = 2.5$ and $a = 0.13$, with initial income distribution having mean \$47101 and median \$35172.

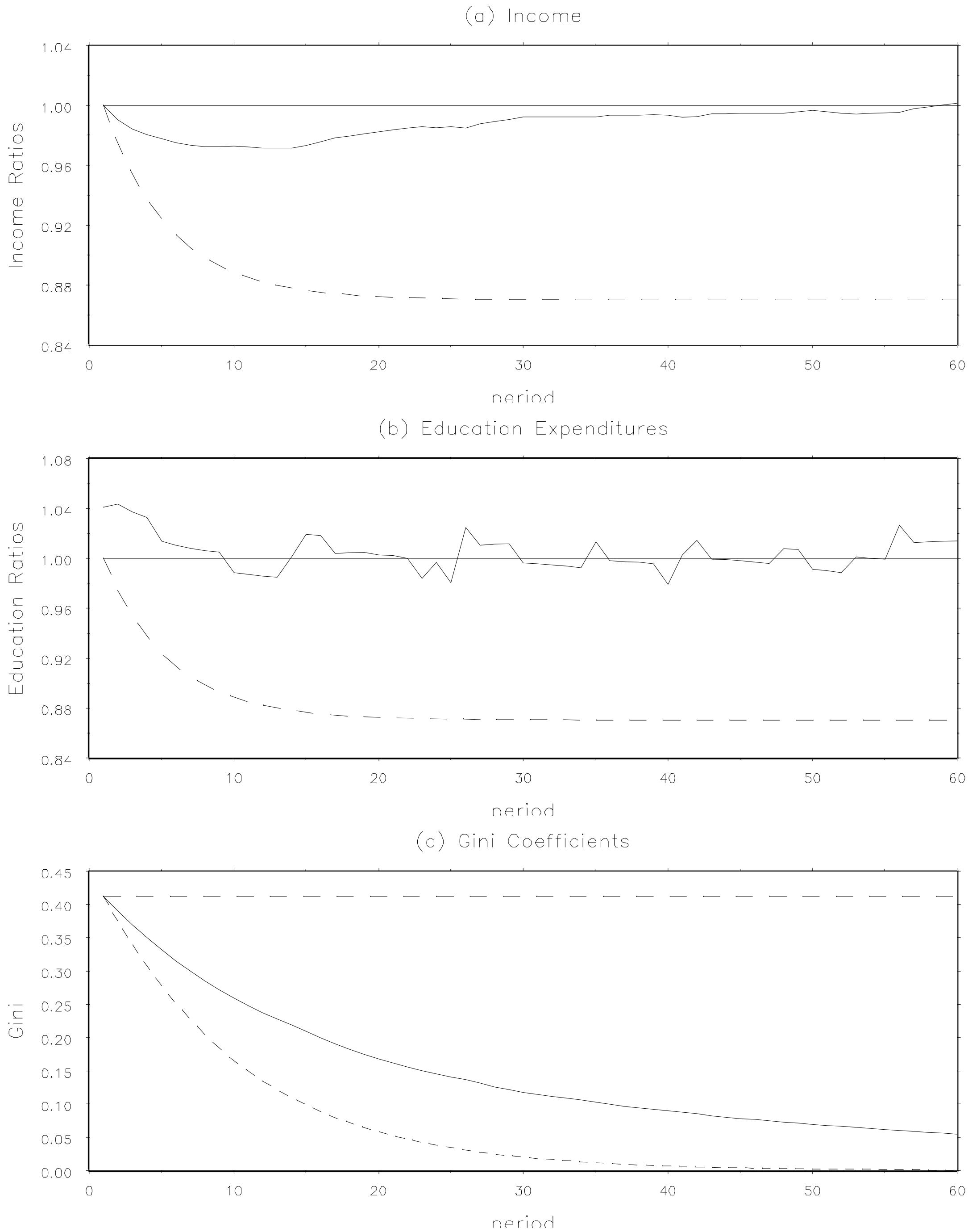

Figure 8: Comparisons of incomes, education expenditures and gini coefficients between the mixed education model and the public and private education benchmark models. In panels (a) and (b), mixed:private model ratios given by solid lines and public:private ratios given by dashed lines with horizontal solid line being private benchmark. The Gini coefficients for the private (dashed line), mixed (solid line) and public (short dashed line) education models are presented in panel (c). Parameter values are $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.065$, with initial income distribution having mean \$47101 and median \$35172.

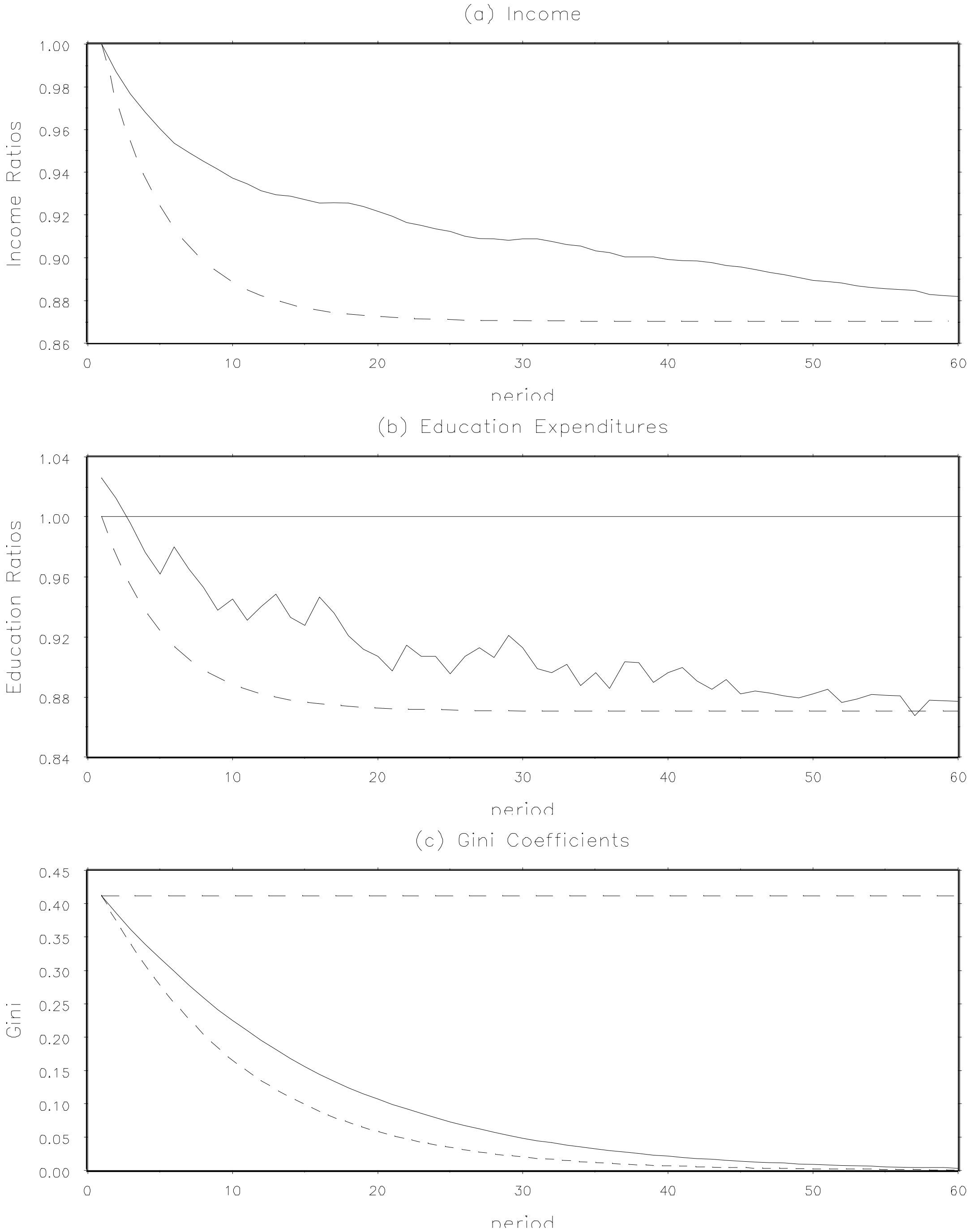

Figure 9: Comparisons of incomes, education expenditures and gini coefficients between the mixed education model and the public and private education benchmark models. In panels (a) and (b), mixed:private model ratios given by solid lines and public:private ratios given by dashed lines with horizontal solid line being private benchmark. The Gini coefficients for the private (dashed line), mixed (solid line) and public (short dashed line) education models are presented in panel (c). Parameter values are $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.26$, with initial income distribution having mean \$47101 and median \$35172.

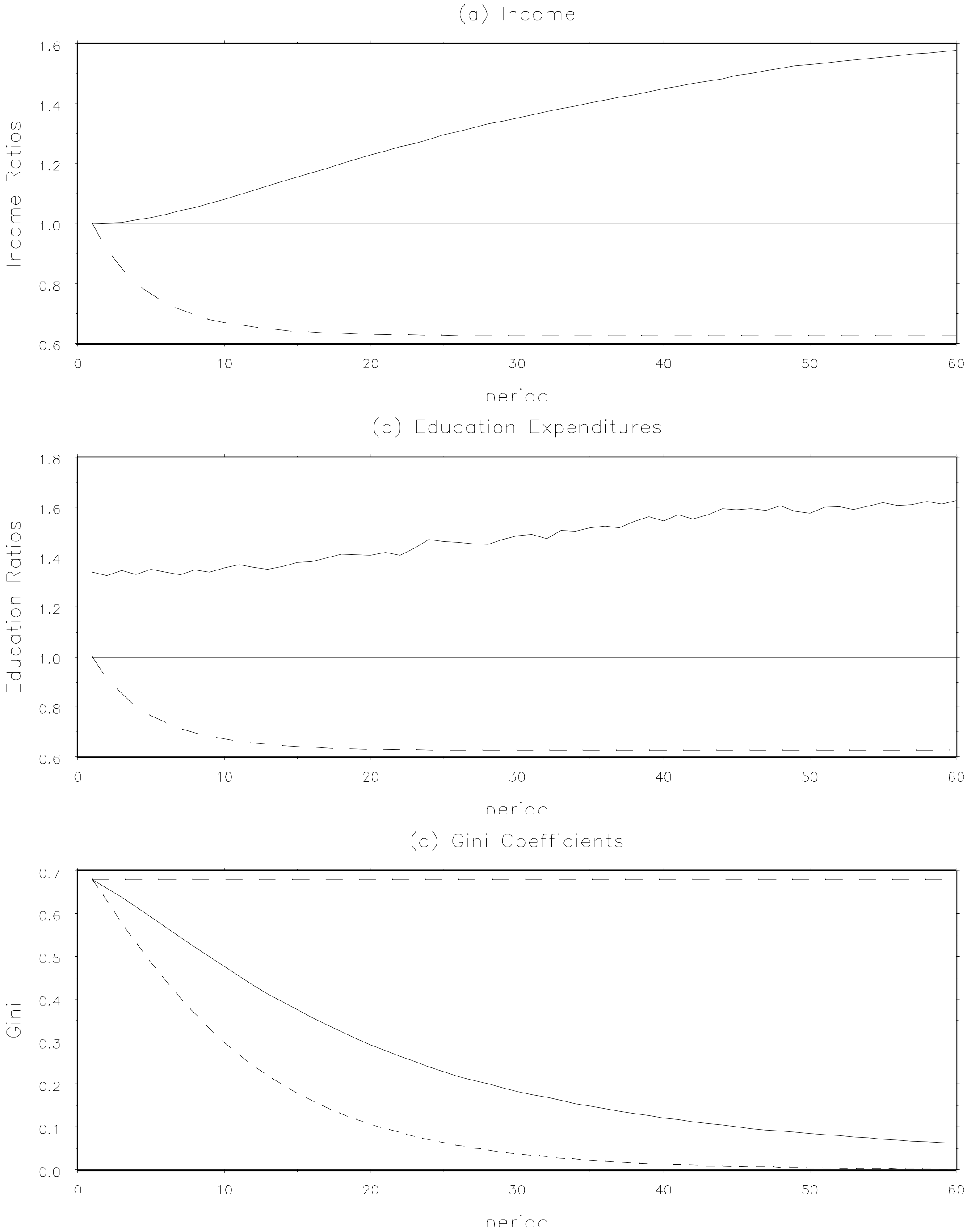

Figure 10: Comparisons of incomes, education expenditures and gini coefficients between the mixed education model and the public and private education benchmark models. In panels (a) and (b), mixed:private model ratios given by solid lines and public:private ratios given by dashed lines with horizontal solid line being private benchmark. The Gini coefficients for the private (dashed line), mixed (solid line) and public (short dashed line) education models are presented in panel (c). Parameter values are $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.13$, with initial income distribution having mean \$94202 and median \$35172.

Table 1: Public education enrolment and tax rate in the education choice model with parameter values $\gamma = 0.9, \theta = 1.6$ and $a = 0.13$ and initial income distribution has mean of \$47101 and median of \$35172.

Period	P_t	Tax, τ_t	Period	P_t	Tax, τ_t
1	0.9417	0.096	31	0.9818	0.112
2	0.9417	0.096	32	0.9820	0.111
3	0.9417	0.096	33	0.9820	0.109
4	0.9421	0.096	34	0.9820	0.108
5	0.9421	0.095	35	0.9821	0.108
6	0.9421	0.095	36	0.9821	0.107
7	0.9441	0.096	37	0.9826	0.107
8	0.9489	0.098	38	0.9831	0.107
9	0.9550	0.102	39	0.9837	0.107
10	0.9560	0.102	40	0.9837	0.106
11	0.9560	0.101	41	0.9840	0.106
12	0.9560	0.100	42	0.9865	0.109
13	0.9570	0.100	43	0.9870	0.109
14	0.9570	0.099	44	0.9874	0.109
15	0.9593	0.100	45	0.9878	0.109
16	0.9658	0.105	46	0.9882	0.109
17	0.9658	0.104	47	0.9894	0.111
18	0.9658	0.103	48	0.9894	0.109
19	0.9666	0.103	49	0.9894	0.109
20	0.9666	0.102	50	0.9894	0.108
21	0.9707	0.105	51	0.9907	0.110
22	0.9715	0.105	52	0.9913	0.111
23	0.9715	0.104	53	0.9915	0.111
24	0.9715	0.103	54	0.9921	0.112
25	0.9758	0.107	55	0.9923	0.112
26	0.9758	0.106	56	0.9923	0.111
27	0.9758	0.105	57	0.9923	0.110
28	0.9783	0.108	58	0.9932	0.112
29	0.9784	0.107	59	0.9935	0.114
30	0.9797	0.108	60	0.9935	0.113

Table 2: Public education enrolment and tax rate in the education choice model with parameter values $\gamma = 0.8, \theta = 1.6$ and $a = 0.13$ and initial income distribution has mean of $47101 and median of $35172.

Period	P_t	Tax, τ_t	Period	P_t	Tax, τ_t
1	0.9417	0.096	31	0.9935	0.111
2	0.9418	0.096	32	0.9940	0.111
3	0.9418	0.095	33	0.9946	0.112
4	0.9418	0.094	34	0.9948	0.111
5	0.9550	0.102	35	0.9957	0.112
6	0.9560	0.101	36	0.9958	0.111
7	0.9560	0.099	37	0.9966	0.112
8	0.9655	0.105	38	0.9968	0.112
9	0.9656	0.103	39	0.9970	0.113
10	0.9661	0.102	40	0.9973	0.113
11	0.9709	0.105	41	0.9976	0.114
12	0.9710	0.103	42	0.9976	0.113
13	0.9753	0.106	43	0.9977	0.113
14	0.9785	0.109	44	0.9979	0.113
15	0.9794	0.108	45	0.9979	0.112
16	0.9818	0.111	46	0.9984	0.113
17	0.9820	0.109	47	0.9985	0.114
18	0.9825	0.108	48	0.9987	0.113
19	0.9831	0.107	49	0.9989	0.113
20	0.9842	0.107	50	0.9990	0.114
21	0.9860	0.108	51	0.9990	0.112
22	0.9874	0.109	52	0.9992	0.113
23	0.9892	0.111	53	0.9993	0.113
24	0.9893	0.109	54	0.9995	0.114
25	0.9900	0.109	55	0.9995	0.114
26	0.9914	0.111	56	0.9995	0.114
27	0.9921	0.112	57	0.9995	0.113
28	0.9922	0.110	58	0.9997	0.114
29	0.9932	0.112	59	0.9997	0.114
30	0.9935	0.113	60	0.9997	0.114

Table 3: Public education enrolment and tax rate in the education choice model with parameter values $\gamma = 0.7, \theta = 2.5$ and $a = 0.13$ and initial income distribution has mean of $47101 and median of $35172.

Period	P_t	Tax, τ_t	Period	P_t	Tax, τ_t
1	0.9417	0.096	31	0.9985	0.113
2	0.9422	0.096	32	0.9987	0.112
3	0.9551	0.103	33	0.9990	0.114
4	0.9565	0.101	34	0.9992	0.114
5	0.9591	0.1	35	0.9993	0.113
6	0.9661	0.104	36	0.9995	0.114
7	0.9705	0.105	37	0.9995	0.114
8	0.9715	0.103	38	0.9997	0.114
9	0.9781	0.109	39	0.9997	0.114
10	0.9798	0.108	40	0.9997	0.114
11	0.9818	0.109	41	0.9997	0.114
12	0.9821	0.107	42	0.9998	0.114
13	0.9837	0.107	43	0.9998	0.114
14	0.9872	0.11	44	0.9999	0.115
15	0.9877	0.108	45	0.9999	0.115
16	0.9892	0.109	46	1.0000	0.115
17	0.9914	0.112	47	1.0000	0.115
18	0.992	0.111	48	1.0000	0.115
19	0.9928	0.111	49	1.0000	0.115
20	0.9935	0.113	50	1.0000	0.115
21	0.994	0.111	51	1.0000	0.115
22	0.9947	0.112	52	1.0000	0.115
23	0.9957	0.112	53	1.0000	0.115
24	0.9965	0.113	54	1.0000	0.115
25	0.9968	0.112	55	1.0000	0.115
26	0.997	0.112	56	1.0000	0.115
27	0.9976	0.114	57	1.0000	0.115
28	0.9977	0.114	58	1.0000	0.115
29	0.9979	0.113	59	1.0000	0.115
30	0.9984	0.114	60	1.0000	0.115

Table 4: Public education enrolment and tax rate in the education choice model with parameter values $\gamma = 0.9, \theta = 1.6$ and $a = 0.065$ and initial income distribution has mean of $47101 and median of $35172.

Period	P_t	Tax, τ_t	Period	P_t	Tax, τ_t
1	0.9395	0.051	31	0.9657	0.054
2	0.9415	0.052	32	0.9657	0.054
3	0.9415	0.052	33	0.9659	0.054
4	0.9415	0.052	34	0.9662	0.054
5	0.9415	0.051	35	0.9705	0.056
6	0.9415	0.051	36	0.9705	0.055
7	0.9415	0.051	37	0.9705	0.055
8	0.9415	0.051	38	0.9705	0.055
9	0.9415	0.051	39	0.9708	0.055
10	0.9415	0.050	40	0.9708	0.054
11	0.9415	0.050	41	0.9734	0.056
12	0.9415	0.050	42	0.9753	0.057
13	0.9415	0.050	43	0.9753	0.056
14	0.9476	0.052	44	0.9753	0.056
15	0.9536	0.054	45	0.9754	0.056
16	0.9545	0.054	46	0.9758	0.056
17	0.9545	0.053	47	0.9759	0.056
18	0.9545	0.053	48	0.9776	0.057
19	0.9546	0.053	49	0.9780	0.057
20	0.9555	0.053	50	0.9780	0.056
21	0.9560	0.053	51	0.9780	0.056
22	0.9570	0.053	52	0.9780	0.056
23	0.9570	0.052	53	0.9791	0.057
24	0.9579	0.053	54	0.9796	0.057
25	0.9579	0.052	55	0.9798	0.057
26	0.9656	0.056	56	0.9818	0.059
27	0.9656	0.055	57	0.9818	0.058
28	0.9656	0.055	58	0.9818	0.058
29	0.9657	0.055	59	0.9819	0.058
30	0.9657	0.054	60	0.9820	0.058

Table 5: Public education enrolment and tax rate in the education choice model with parameter values $\gamma = 0.9, \theta = 1.6$ and $a = 0.26$ and initial income distribution has mean of $47101 and median of $35172.

Period	P_t	Tax, τ_t	Period	P_t	Tax, τ_t
1	0.9552	0.182	31	0.9935	0.200
2	0.9568	0.183	32	0.9940	0.200
3	0.9570	0.182	33	0.9947	0.202
4	0.9570	0.180	34	0.9948	0.199
5	0.9572	0.179	35	0.9958	0.202
6	0.9657	0.189	36	0.9960	0.200
7	0.9658	0.187	37	0.9968	0.205
8	0.9665	0.186	38	0.9970	0.205
9	0.9669	0.184	39	0.9970	0.202
10	0.9712	0.189	40	0.9975	0.204
11	0.9717	0.187	41	0.9976	0.205
12	0.9758	0.192	42	0.9976	0.203
13	0.9785	0.196	43	0.9977	0.202
14	0.9785	0.193	44	0.9979	0.204
15	0.9797	0.193	45	0.9980	0.202
16	0.9820	0.199	46	0.9984	0.203
17	0.9824	0.197	47	0.9985	0.203
18	0.9825	0.194	48	0.9987	0.203
19	0.9834	0.193	49	0.9989	0.203
20	0.9843	0.193	50	0.9990	0.204
21	0.9852	0.192	51	0.9992	0.205
22	0.9879	0.198	52	0.9992	0.203
23	0.9885	0.197	53	0.9994	0.204
24	0.9895	0.198	54	0.9995	0.205
25	0.9898	0.196	55	0.9995	0.205
26	0.9915	0.200	56	0.9995	0.205
27	0.9922	0.202	57	0.9995	0.202
28	0.9927	0.201	58	0.9997	0.205
29	0.9935	0.205	59	0.9997	0.205
30	0.9935	0.203	60	0.9997	0.205

Table 6: Public education enrolment and tax rate in the education choice model with parameter values $\gamma = 0.9, \theta = 1.6$ and $a = 0.13$ and initial income distribution has mean of $94202 and median of $35172.

Period	P_t	Tax, τ_t	Period	P_t	Tax, τ_t
1	0.9233	0.103	31	0.9717	0.109
2	0.9235	0.102	32	0.9717	0.107
3	0.9286	0.106	33	0.9741	0.110
4	0.9286	0.104	34	0.9742	0.109
5	0.9318	0.107	35	0.9757	0.110
6	0.9318	0.105	36	0.9761	0.110
7	0.9318	0.103	37	0.9764	0.109
8	0.9362	0.106	38	0.9778	0.111
9	0.9366	0.104	39	0.9785	0.112
10	0.9397	0.106	40	0.9785	0.110
11	0.9421	0.107	41	0.9798	0.112
12	0.9423	0.105	42	0.9798	0.110
13	0.9424	0.103	43	0.9805	0.111
14	0.9456	0.104	44	0.9817	0.113
15	0.9496	0.106	45	0.9817	0.112
16	0.9513	0.106	46	0.9819	0.112
17	0.9536	0.107	47	0.9820	0.111
18	0.9556	0.108	48	0.9825	0.112
19	0.9564	0.107	49	0.9825	0.110
20	0.9570	0.106	50	0.9826	0.109
21	0.9596	0.107	51	0.9837	0.111
22	0.9597	0.105	52	0.9841	0.111
23	0.9634	0.108	53	0.9843	0.110
24	0.9657	0.111	54	0.9853	0.111
25	0.9658	0.109	55	0.9860	0.112
26	0.9660	0.108	56	0.9862	0.111
27	0.9665	0.107	57	0.9866	0.111
28	0.9670	0.106	58	0.9872	0.112
29	0.9696	0.108	59	0.9873	0.111
30	0.9708	0.109	60	0.9878	0.112

Appendix

A1 Proofs

Proof of Proposition 1.

The income threshold, $y^*_{t+1}\left(E^*_{t+1},\tau^*_{t+1}\right)$, given by equation (14), is found by equating (10) and (12), substituting (7), (8), (9) and (11) and replacing $\left(E_{t+1},\tau_{t+1}\right)$ and $y^*_{i,t+1}$ with $\left(E^*_{t+1},\tau^*_{t+1}\right)$ and y^*_{t+1}.

Applying standard rules of logarithms leads to the following result:

$$\ln\left(\frac{a\left(1-\tau^*_{t+1}\right)y^*_{t+1}}{\left(1+a\right)E^*_{t+1}}\right)^a=\ln\left(1+a\right) \tag{27}$$

taking the exponential and rearranging for A^*_t:

$$y^*_{t+1}=\frac{E^*_{t+1}\left(1+a\right)^{\frac{1+a}{a}}}{\left(1-\tau^*_{t+1}\right)a} \tag{28}$$

which is the desired result.

Parts (i) and (ii) of the proposition are proved by considering the difference between equations (10) and (12) at $\left(E^*_{t+1},\tau^*_{t+1}\right)$:

$$D_{i,t+1}=V^u\left(E^*_{t+1},\tau^*_{t+1};y_{i,t+1}\right)-V^r\left(\tau^*_{t+1};y_{i,t+1}\right) \tag{29}$$

Showing the sign of $D_{i,t+1}$ to change at the boundary values for income, $y_{i,t+1}=\{0,\infty\}$, and that the slope of $D_{i,t+1}$ is monotonic in $y_{i,t+1}$ yields the results in parts (i) and (ii) of the proposition.

Substituting (10), (12), (7), (8), (9) and (11) into (29) and differentiating with respect to $y_{i,t+1}$, the result is:

$$\frac{\partial D_{i,t+1}}{\partial y_{i,t+1}}=-\frac{a}{y_{i,t+1}}<0 \tag{30}$$

Thus, $D_{i,t+1}$ is decreasing in $y_{i,t+1}$. This means that as income rises, equilibrium private education utility rises faster than equilibrium public education utility.

The value of $D_{i,t+1}$ at the boundaries are found by taking the limits.

$$\begin{aligned}\lim_{y_{i,t+1}\to 0}D_{i,t+1} &= \lim_{y_{i,t+1}\to 0}\left[a\ln\left(\frac{\left(1+a\right)^{\frac{1+a}{a}}\left(E^*_{t+1}\right)}{a\left(1-\tau^*_{t+1}\right)y_{i,t+1}}\right)\right] \\ &= +\infty\end{aligned} \tag{31}$$

As $y_{i,t+1}\to 0$, equation (31) diverges to $+\infty$. Thus the difference $D_{i,t+1}$ is positive at the lower boundary for $y_{i,t+1}$. This means that at very low incomes, agents are much better off in public schools.

Taking the limit as $y_{i,t+1} \to \infty$:

$$\begin{aligned} \lim_{y_{i,t+1}\to\infty} D_{i,t+1} &= \lim_{y_{i,t+1}\to\infty} \left[a \ln \left(\frac{(1+a)^{\frac{1+a}{a}} \left(E^*_{t+1}\right)}{a\left(1-\tau^*_{t+1}\right) y_{i,t+1}} \right) \right] \\ &= -\infty \end{aligned} \tag{32}$$

In this case, as $y_{i,t+1} \to \infty$, the logarithm in equation (32) is operating on a number converging to zero which leads to $\lim\limits_{y_{i,t+1}\to\infty} D_{i,t+1} = -\infty$. This means that at very high incomes, agents are much better off in private schools.

Thus the difference, $D_{i,t+1}$, is positive for small values of $y_{i,t+1}$, negative for large values of $y_{i,t+1}$ and monotonically decreasing in $y_{i,t+1}$ when $y_{i,t+1} > 0$, which means that $D_{i,t+1}$ can have only one root in $\mathbb{R}_{++}$. Since by construction, $D_{i,t+1} = 0$ when $y_{i,t+1} = y^*_{t+1}$, then the threshold inheritance level $y^*_{t+1}\left(E^*_{t+1}, \tau^*_{t+1}\right)$ is unique. This proves part (i).

Using the fact that $D_{i,t+1} = V^u - V^r > 0$ for $y_{i,t+1} < y^*_{t+1}$, agents with $y_{i,t+1} < y^*_{t+1}$ will have higher utility consuming public rather than private education. Similarly, $D_{i,t+1} = V^u - V^r < 0$ for $y_{i,t+1} > y^*_{t+1}$, agents with $y_{i,t+1} > y^*_{t+1}$ will have higher utility when consuming private education. When $y_{i,t+1} = y^*_{t+1}$, $D_{i,t+1} = V^u - V^r = 0$ so agents are indifferent between the two systems, but it has been assumed that in this case the agent will prefer and choose public education. Thus part (ii) of the proposition is proved.

■

Proof of Proposition 2

Proposition 2 follows from a comparison of equations (20) and (21). We are looking for cases where:

$$h^u_{i,t+1} > h^r_{i,t+1} \tag{33}$$

which by substitution gives us:

$$\theta \left(\frac{\tau_t H_t}{P_t} \right)^{1-\gamma} h^{\gamma}_{i,t} > \theta \left(\frac{a\left(1-\tau_{t+1}\right)}{(1+a)} \right)^{1-\gamma} h_{i,t} \tag{34}$$

After some manipulation, (?) is equivalent to:

$$\left(\frac{a\left(1-\tau_{t+1}\right)}{(1+a)} \right) h_{i,t} < \left(\frac{\tau_t H_t}{P_t} \right) \tag{35}$$

Recalling that $E_t = \left(\frac{\tau_t H_t}{P_t}\right)$ and with some algebraic manipulation:

$$h_{i,t} < \frac{E_t\,(1+a)}{a\left(1-\tau_{t+1}\right)} = \widehat{h}_t \tag{36}$$

Thus (33) holds when (36) holds and the proposition is proved.

■

Proof of Corollary 1

Recall that

$$\widehat{y}_{t+1} = \widehat{h}_t = \frac{E_t(1+a)}{a(1-\tau_{t+1})} \quad (37)$$

$$y^*_{t+1} = h^*_t = \frac{E_{t+1}(1+a)^{\frac{1+a}{a}}}{a(1-\tau_{t+1})} \quad (38)$$

We can rewrite (38) as

$$y^*_{t+1} = h^*_t = \frac{E_{t+1}(1+a)}{a(1-\tau_{t+1})}(1+a)^{\frac{1}{a}}$$

and because $a \in (0,1)$, a simple comparison with (37) shows that $y^*_{t+1} > \widehat{y}_{t+1}$ and the result is proved.

■

A2 Solution Algorithm

The model is solved by defining a grid over the feasible tax range and calculating indirect preferences over the tax rate numerically in order to determine the political and economic equilibrium. This approach is used by Glomm and Ravikumar (1998). An alternative solution technique is employed by Epple and Romano (1996), which solves the first order conditions in search of a general equilibrium. In the dynamic setting studied here, such a technique is not feasible because the closed form of the income distribution is required, since both the cdf and pdf of income enter the first order conditions. In the model in this paper, closed forms for the distribution of income are not available except for the first generation which is assumed to be lognormal. Thus, solving all the first order conditions in search of an equilibrium is not feasible.

The steps involved in the algorithm are as follows.

1. The income endowments of the first generation of the sample population are randomly drawn. This is done using a standard normal random number generator, the numbers are subsequently transformed into a lognormal distribution, calibrated in the manner described in the Section 6 of the paper. Parameters are chosen and a tax grid is defined over the unit interval,with 1000 points found to be adequate.

2. The problem reduces to the optimal allocation of income between consumption and education expenditure on the agent's child, given the public and private education

alternatives. At each value on the tax grid, the income threshold where agents are indifferent between public and private education is calculated. This is done by proposing a value for P_{t+1}, the proportion of the population in public schools, and calculating the threshold y_{t+1}^* that corresponds with the proposed P_{t+1}, according to equation (14). Using the distribution of income, the actual proportion of the population with incomes below y_{t+1}^* is calculated, given by equation (15), and compared with the proposed P_{t+1}. If these two values are equal, the correct solution has been found, otherwise a new proposed solution for P_{t+1} is made and the process is repeated until a value for P_{t+1} consistent with equations (14) and (15) is found.

3. At each point on the tax grid, given the indifference income level $\left(y_{t+1}^*\right)$ and the proportion of public school users (P_{t+1}), consumption and public and private education expenditure choices are calculated. Using these values each agent's utility at each point on the tax grid is also calculated to generate indirect preferences over tax rates. A political equilibrium is the tax rate which is preferred by a majority and cannot be defeated by any other tax proposal. This is easily identified as the most preferred tax rate of the median income agent, given Result 1.

4. Given the political equilibrium, $\left(E_{t+1}^*, \tau_{t+1}^*\right)$, the corresponding income threshold, $y_{t+1}^*\left(E_{t+1}^*, \tau_{t+1}^*\right)$, and public education enrolment $\left(P_{t+1}^*\right)$ are determined. This in turn yields the equilibrium education decisions of every parent and the incomes of their children, given by equation (7), when they are old and ready to make an allocation between consumption and the education of their own child. Thus the next generation is ready to allocate their income between consumption and education which can be determined by returning to step 2 and repeating steps 2, 3 and 4.

5. The process is repeated until the desired time horizon is solved for.

A3 Figures

The following figures are provided for the benefit of the reader/referee. They are not intended for final publication and will be available from the author upon request.

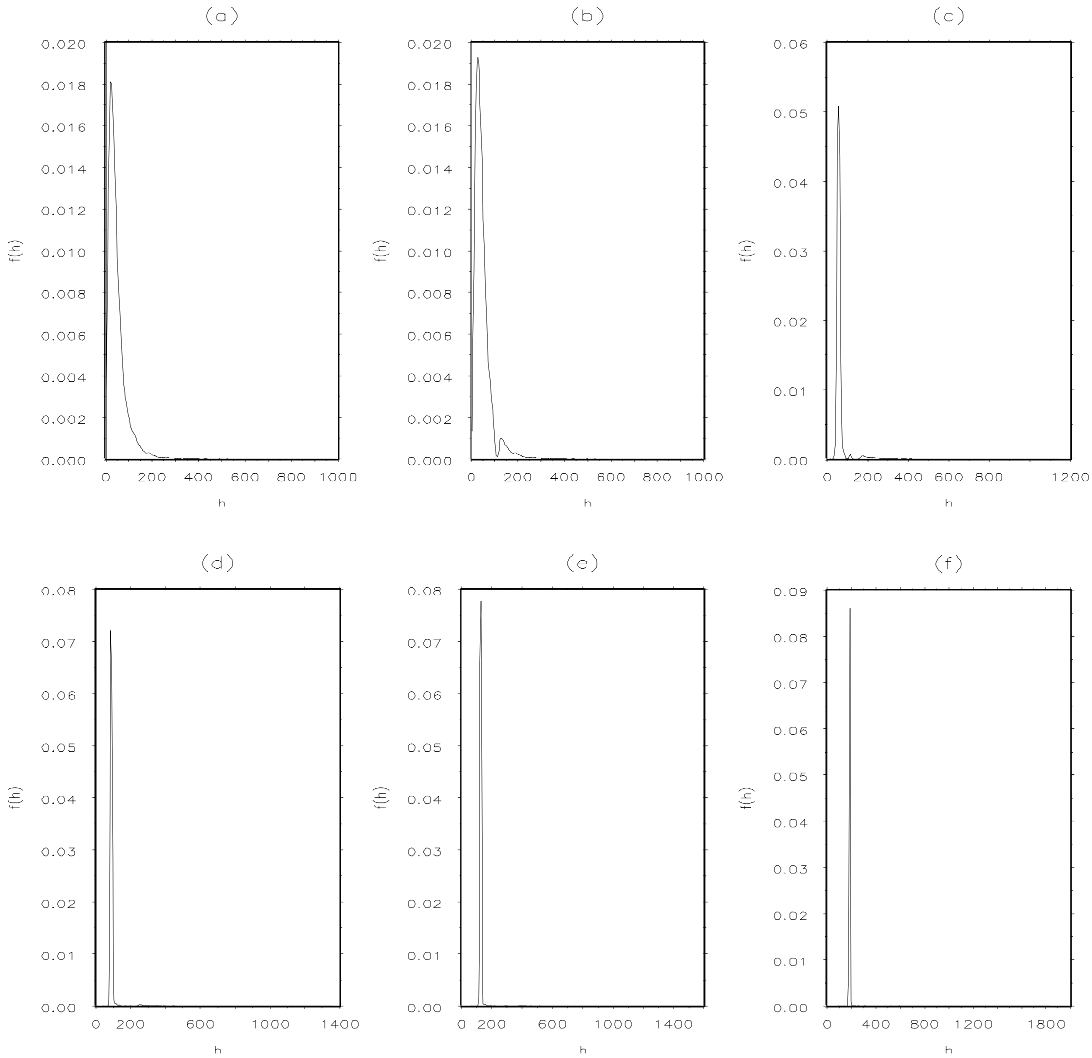

Figure A1: Income Distribution in the model of education choice. Panel (a) provides the initial distribution described above. Panels (b), (c), (d), (e) and (f) provide the endogenous income distributions after 1, 10, 20, 30 and 40 generations respectively, where human capital (income) is measured in thousands of dollars. Parameter values are $\gamma = 0.8$, $\theta = 1.6$ and $a = 0.13$, with initial income distribution having mean \$47101 and median \$35172.

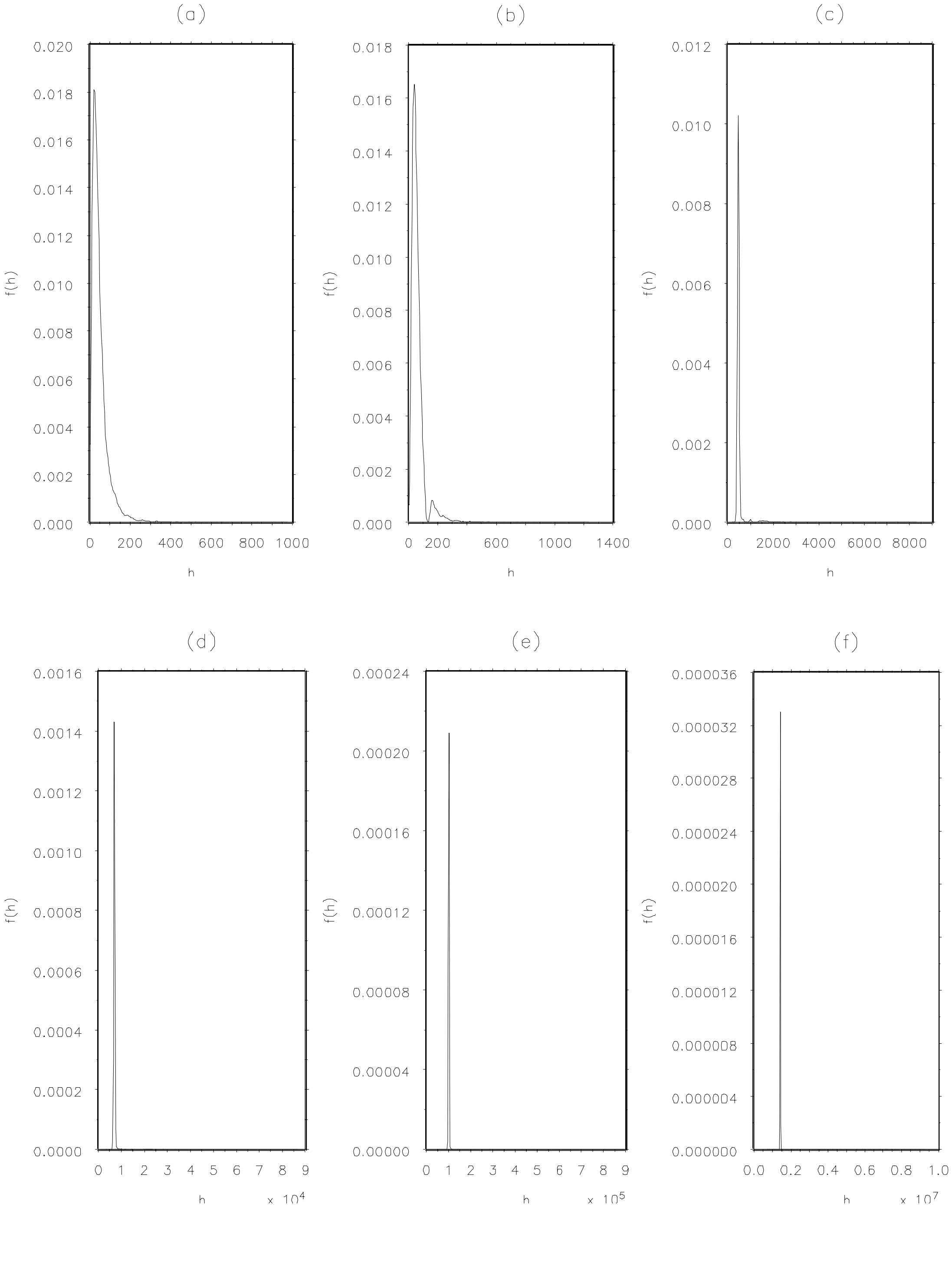

Figure A2: Income Distribution in the model of education choice. Panel (a) provides the initial distribution described above. Panels (b), (c), (d), (e) and (f) provide the endogenous income distributions after 1, 10, 20, 30 and 40 generations respectively, where human capital (income) is measured in thousands of dollars. Parameter values are $\gamma = 0.7$, $\theta = 2.5$ and $a = 0.13$, with initial income distribution having mean \$47101 and median \$35172.

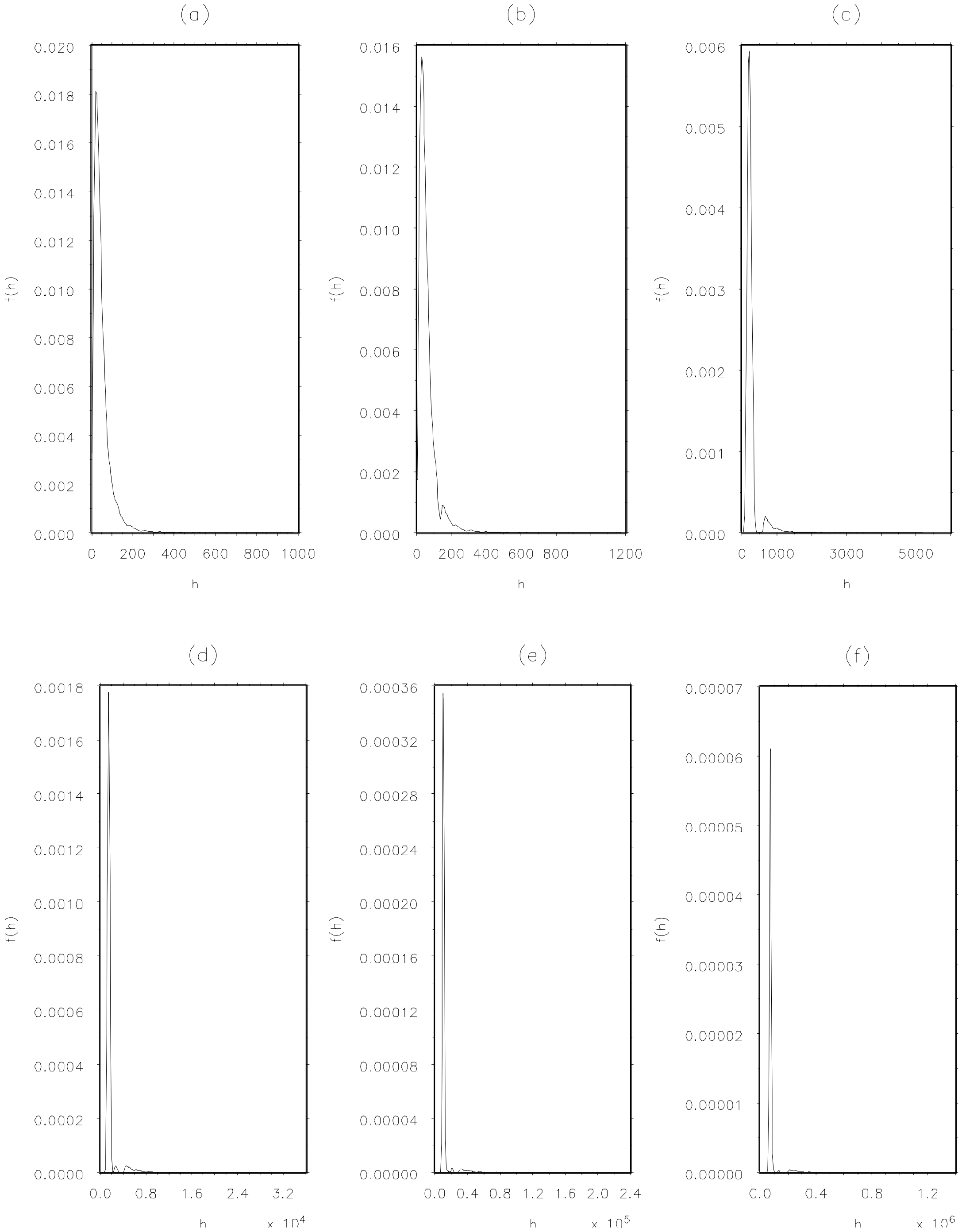

Figure A3: Income Distribution in the model of education choice. Panel (a) provides the initial distribution described above. Panels (b), (c), (d), (e) and (f) provide the endogenous income distributions after 1, 10, 20, 30 and 40 generations respectively, where human capital (income) is measured in thousands of dollars. Parameter values are $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.065$, with initial income distribution having mean \$47101 and median \$35172.

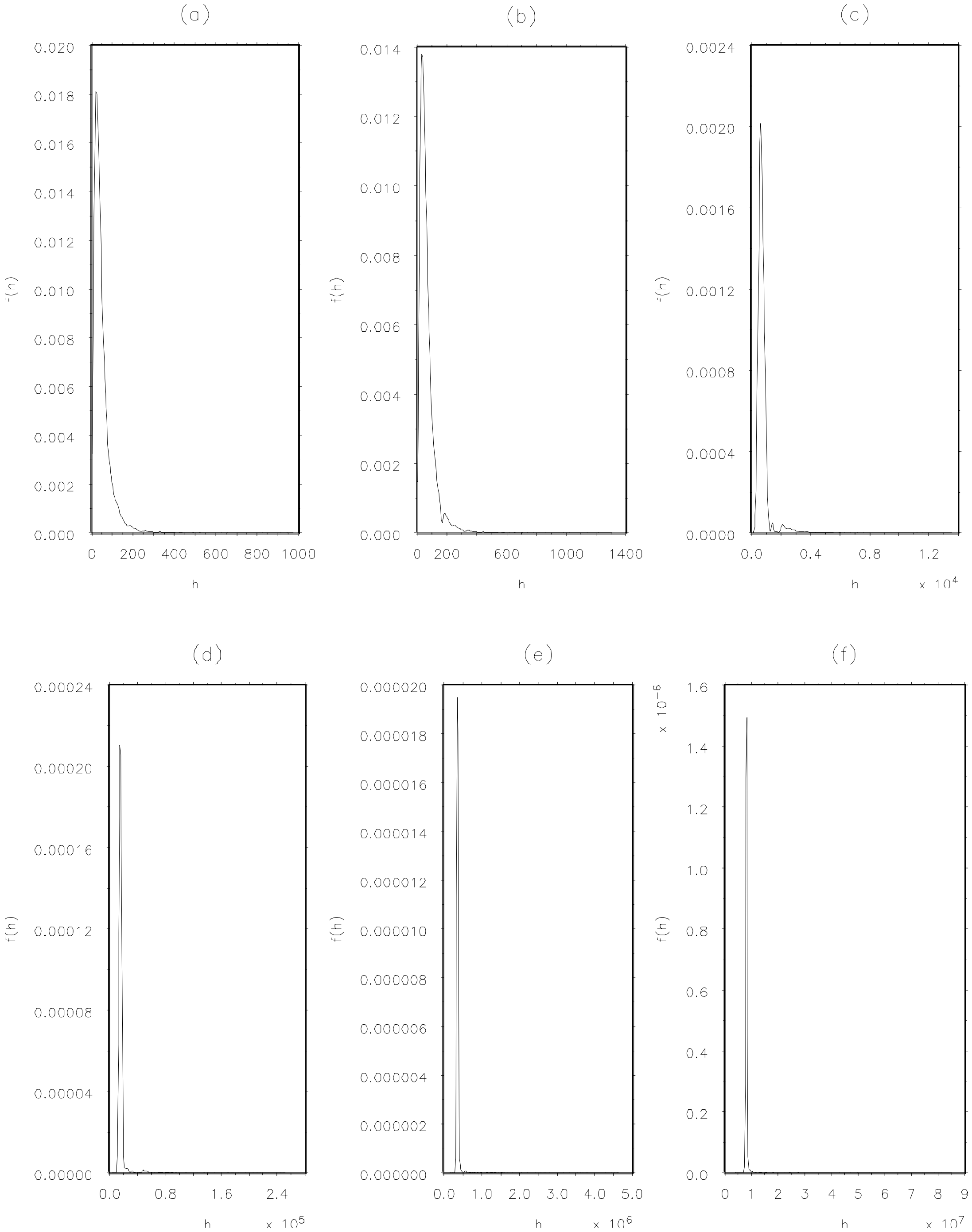

Figure A4: Income Distribution in the model of education choice. Panel (a) provides the initial distribution described above. Panels (b), (c), (d), (e) and (f) provide the endogenous income distributions after 1, 10, 20, 30 and 40 generations respectively, where human capital (income) is measured in thousands of dollars. Parameter values are $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.26$, with initial income distribution having mean \$47101 and median \$35172.

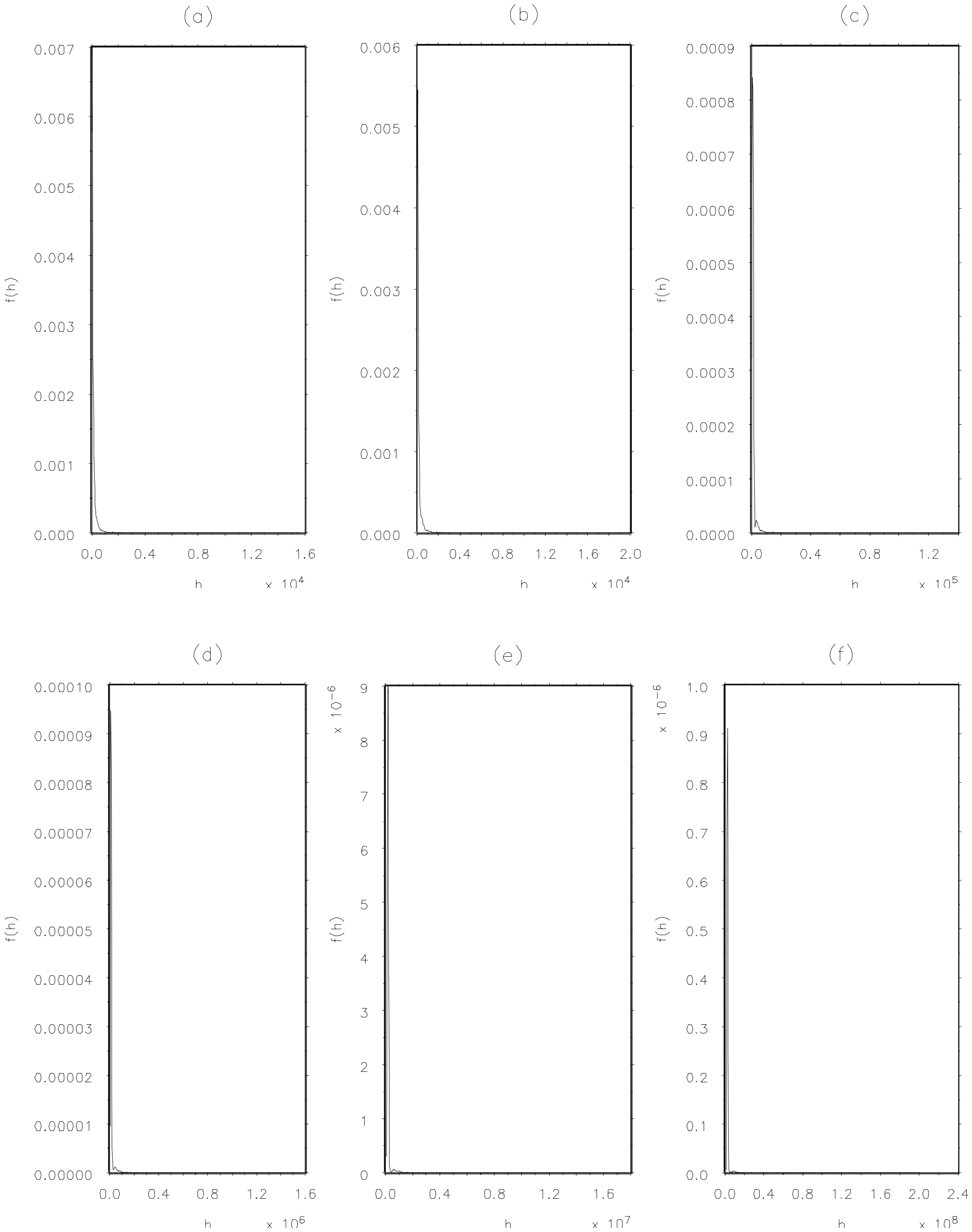

Figure A5: Income Distribution in the model of education choice. Panel (a) provides the initial distribution described above. Panels (b), (c), (d), (e) and (f) provide the endogenous income distributions after 1, 10, 20, 30 and 40 generations respectively, where human capital (income) is measured in thousands of dollars. Parameter values are $\gamma = 0.9$, $\theta = 1.6$ and $a = 0.13$, with initial income distribution having mean \$94202 and median \$35172.